L.2.2005-233
University of Alberta Library
0 1620 3452382 7

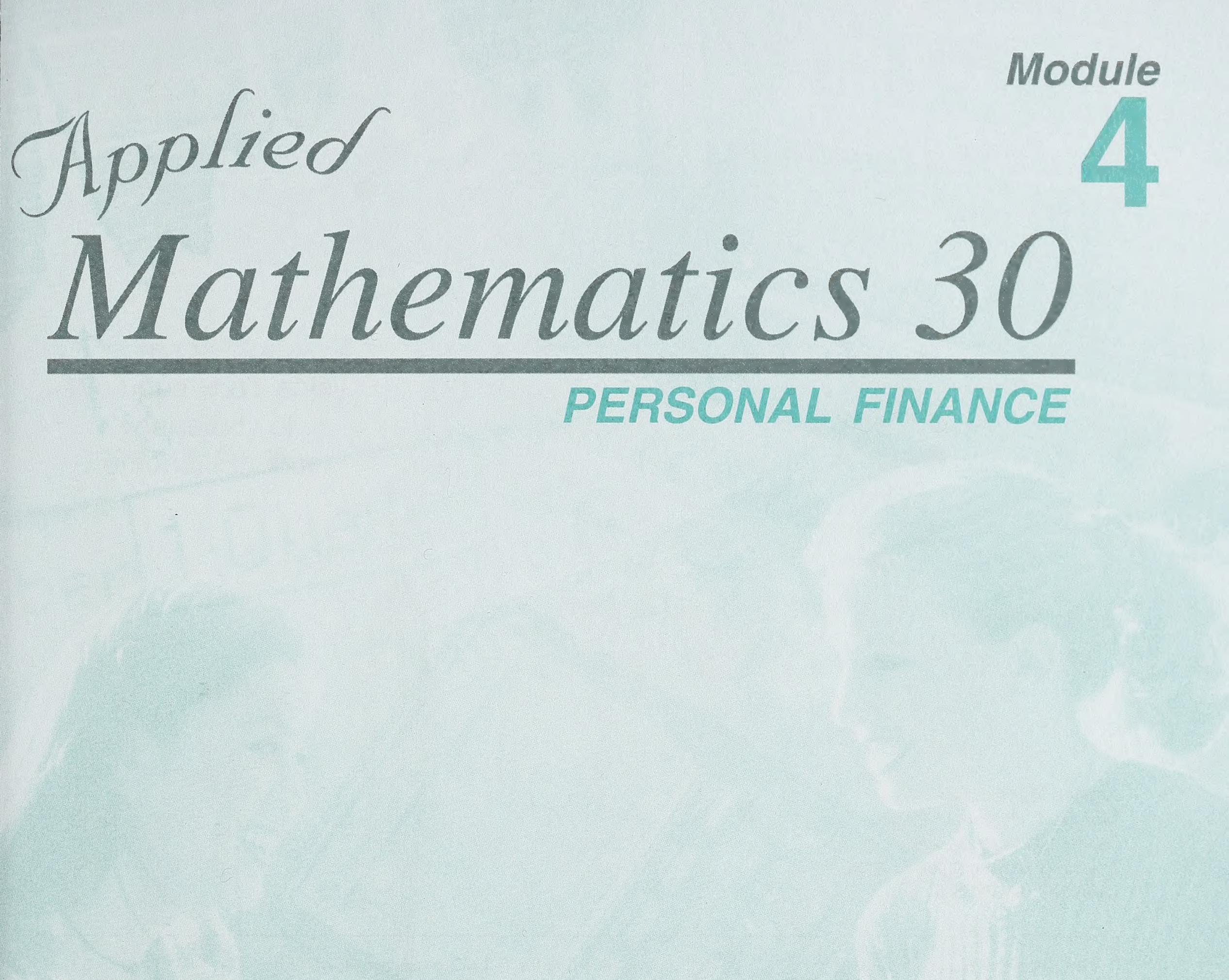
Module
4

Applied Mathematics 30

PERSONAL FINANCE

Learning Technologies Branch

Alberta
LEARNING



Module

4

Applied Mathematics 30

PERSONAL FINANCE

Learning
Technologies
Branch

Alberta
LEARNING

Applied Mathematics 30
Module 4: Personal Finance
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-2228-5

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

ALL RIGHTS RESERVED

Copyright © 2002, the Crown in Right of Alberta, as represented by the Minister of Learning, Alberta Learning, 10155 – 102 Street, Edmonton, Alberta T5J 4L5. All rights reserved. Additional copies may be obtained from the Learning Resources Centre.

No part of this courseware may be reproduced in any form, including photocopying (unless otherwise indicated), without the written permission of Alberta Learning.

Every effort has been made both to provide proper acknowledgement of the original source and to comply with copyright law. If cases are identified where this effort has been unsuccessful, please notify Alberta Learning so that appropriate corrective action can be taken.

IT IS STRICTLY PROHIBITED TO COPY ANY PART OF THESE MATERIALS UNDER THE TERMS OF A LICENCE FROM A COLLECTIVE OR A LICENSING BODY.

UNIVERSITY LIBRARY
UNIVERSITY OF ALBERTA

Welcome

Applied Mathematics 30

Welcome to Module 4.
We hope you'll enjoy
your study of
Personal Finance.

- Module 1: Probability
- Module 2: Matrices
- Module 3: Statistics
- **Module 4: Personal Finance**
- Module 5: Sinusoidal Data
- Module 6: Patterns
- Module 7: Vectors

Applied Mathematics 30 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

Contents

Introduction to Applied Mathematics 30 1

Module Overview
Assessment 9

Module Project: Lease or Buy a Car?
Beginning the Project 10

Activity 1: Investing Money 12

Activity 2: Investing for the Future 21

Activity 3: Investment Portfolios 31

Activity 4: Leasing 34

Activity 5: Mortgages 43

Activity 6: Renting Versus Buying 50

Module Review
Enrichment 54

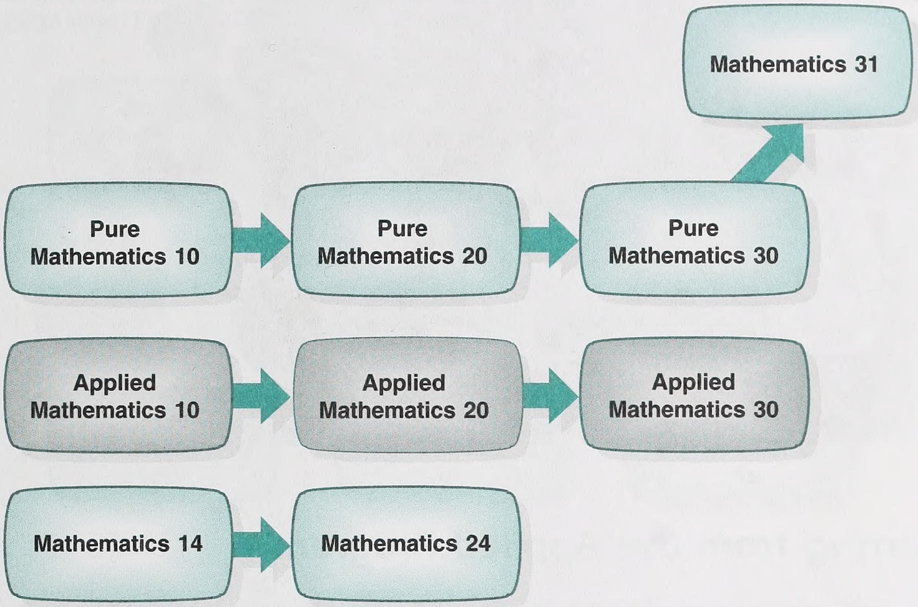
Module Project: Lease or Buy a Car?
Completing the Project 56

Module Summary 58

Appendix
Glossary 60
Calculator Functions 60
Suggested Answers 61
Image Credits 127

Introduction to Applied Mathematics 30

Applied Mathematics 30 is the third course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different career plans. For example, Pure Mathematics 30 is a prerequisite for admission to many university programs. Many colleges and technical institutes, however, will admit students who have successfully completed Applied Mathematics 30.

You may find it helpful to read any of the documents under the heading "New Senior High School Mathematics Update/Post-Secondary Studies Update" at the following Internet site:

http://www.learning.gov.ab.ca/k_12/curriculum/bySubject/math

Before enrolling in Applied Mathematics 30, it is recommended that you talk with a school counsellor about your career plans.

Transferring from the Applied Program

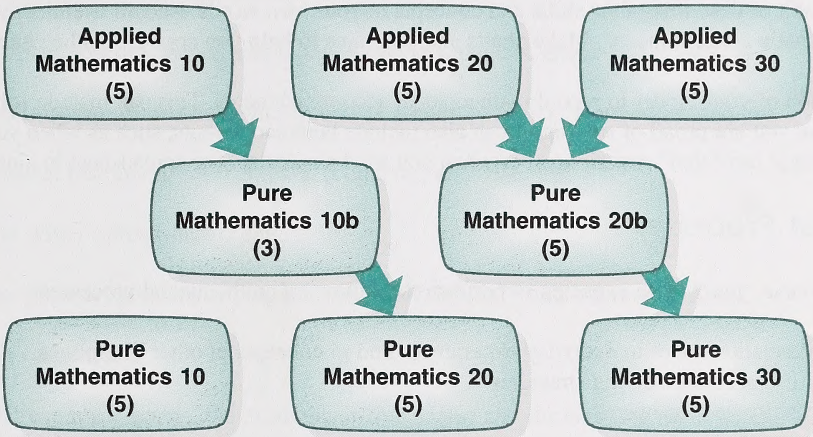
You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

The following table shows some common and independent topics.

Applied Topics	Common Topics	Pure Topics
• linear programming	• spreadsheets	• irrational numbers
• data tables and trends	• line segments and linear graphs	• exponents
• design and layout	• scaling	• polynomial and rational expressions
• metric and imperial measure	• triangles	• mathematical expectations
• data presentation	• financial mathematics	• growth patterns
• vectors and matrices	• quadratic functions	• linear and non-linear systems
• periodic, fractal, and recursive patterns	• circle geometry	• operations on functions
• financial decision making	• the bell curve	• mathematical reasoning
• costing and design problems		• exponential and logarithmic functions
		• conics
		• combinations
		• trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics.

If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may have to take a three-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20 or Applied Mathematics 30, you may have to take a five-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.

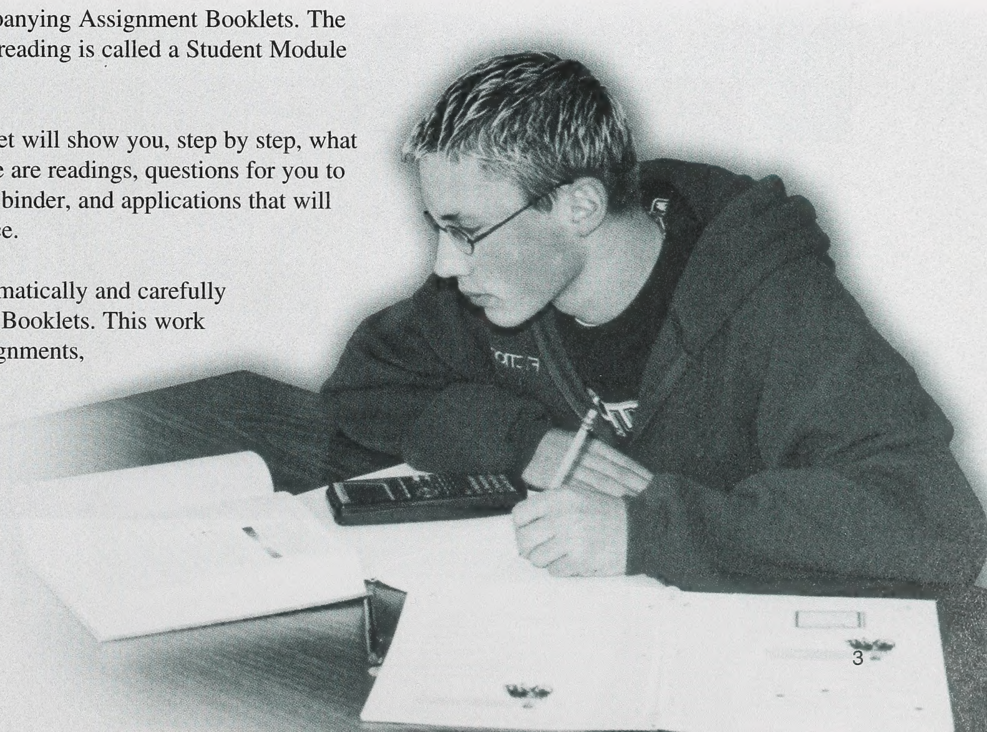


Strategies for Completing Applied Mathematics 30

For each module in Applied Mathematics 30, there is a Student Module Booklet and accompanying Assignment Booklets. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the assignments, projects, and final test.



Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also store your marked assignments here.

- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.

- Keep a section of your binder to record new skills and concepts, as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words. Record useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.

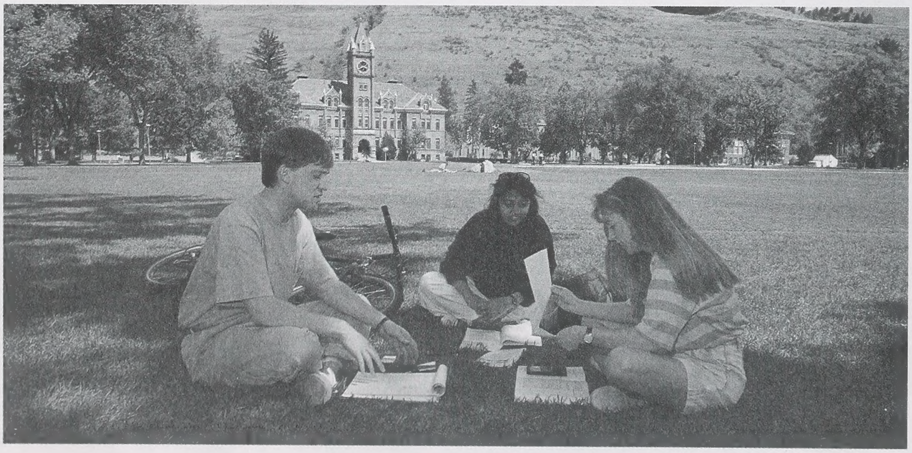
- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an "aha!" experience), or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 30 to be your study partner. You will find that having a friend to discuss mathematical ideas with will make your studying more enjoyable.



Resources You Will Need

In addition to the course materials for Applied Mathematics 30, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 12 Source Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2002)
- the *Addison-Wesley Applied Mathematics 12 Project Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2002)
- a binder, lined loose-leaf paper, graph paper, dividers, pencils, eraser
- metric and imperial measuring devices, such as a ruler, yardstick, metre-stick, and tape measure
- a mathematical instrument set (compass, protractor, and triangles)
- a computer with a spreadsheet program

  Note: Two popular spreadsheet programs are *ClarisWorks*™ and Microsoft® *Excel*.

- a graphing calculator

  Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Texas Instruments	Sharp	Casio	Hewlett-Packard
TI-83	EL-9600C	Algebra FX 2.0	HP 39g †
TI-83 Plus	EL-9600*	CFX-9850 GA-Plus*	
TI-86	EL-9200*	CFX-9850 G*	
TI-89	EL-9300*	CFX-9800 G*	
TI-92*		FX-9700 series*	
TI-92 Plus			

*no longer commercially available

† The HP 39g calculator will remain on the approved list for the 2001–2003 school years and will then be deleted from the approved list.

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you obtain the video program *The TI-83 Graphing Calculator Video Tutor*.

Many of the resources you will need may be purchased locally or from the Learning Resources Centre (LRC). Following is the LRC website:

http://www.lrc.learning.gov.ab.ca

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

Visual Cues

You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the following explanations to discover what the various icons prompt you to do.

Refer to the textbook or the Project Book.

Work with a computer.

Refer to the Applied Mathematics 30 CD.

Contact your teacher for additional information.

Explore the Internet.

Complete specified questions in the Assignment Booklet.

Remember: Any Internet website address given in this module is subject to change.

Where Can I Obtain Diploma Examination Information?

Alberta students will write a diploma examination at the end of the course. Alberta Learning provides several documents to help students prepare for this examination. These documents are found under the heading "Diploma Examinations" at the following Alberta Learning website:

http://www.learning.gov.ab.ca/k_12/testing

Information like course expectations, the makeup of the diploma examination, keyed copies of previous examinations, preparation guides, and calculator policies are available to students at this site.

Each year, in February and September, Alberta Learning provides teachers with information on a **student project**, which teachers **may** use as part of your overall assessment. Information to students will also be posted on the Alberta Learning website. Check with your teacher to determine what you will be expected to do. Be aware that one of the diploma examination's written-response questions will deal with elements of this project and is worth 10% of your diploma examination mark.

You should take advantage of the many sources of information about Applied Mathematics 30. Your success depends on your understanding of course expectations and evaluation procedures. Work closely with your teacher and do not hesitate to ask questions.

Remember, take the initiative to find out all you can about Applied Mathematics 30.

MODULE OVERVIEW

What are your plans for the future? Which career path do you intend to follow when you graduate? Does this career involve a college education or technical training? Will you have to repay student loans when you complete your formal education? What income can you expect to earn? Will that income be sufficient to provide a comfortable living for yourself or your family? Will you buy your own home or rent an apartment? Will you buy or lease your car? How will you invest your savings? How large will your investment portfolio be when you are ready for retirement?

Part of the price of independence is making wise financial decisions. In this module, you will explore some of the tools you can use to help you make these decisions. You will investigate the power of compound interest and the importance of investing regularly, discover how to shelter your income by contributing to an RRSP, discuss the relative benefits of leasing and buying a car, be shown how to calculate mortgage payments, and determine whether renting or buying a home better suits different investment strategies.

Throughout your investigation you will use your graphing calculator and a spreadsheet program quite extensively.

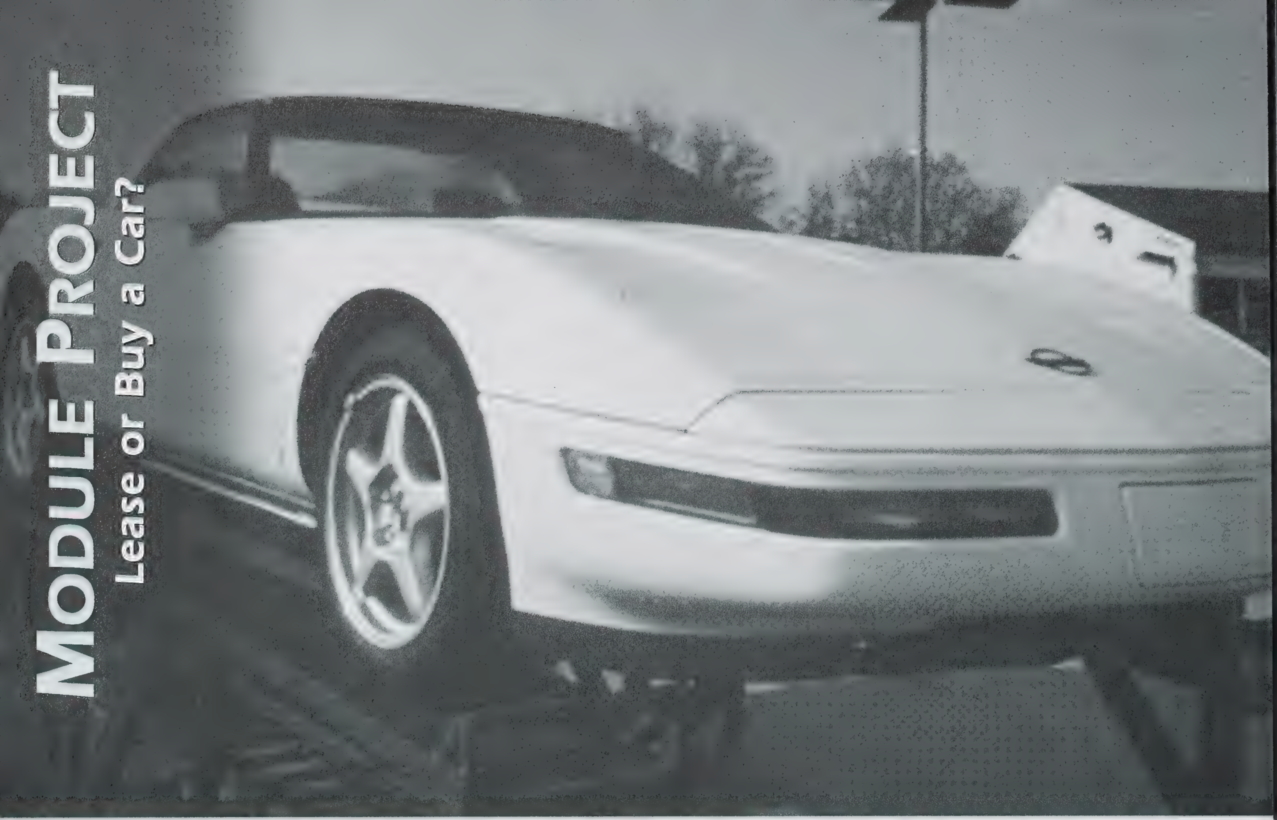
Assessment

Accompanying this Student Module Booklet are two Assignment Booklets. Your grading in this module will be based upon the assignments you submit for assessment. The mark distribution is as follows:

Assignment Booklet 4A	
Activities 1 to 4 Assignment	**90 marks**
Assignment Booklet 4B	
Activities 5 and 6 Assignment	**55 marks**
Module Review Assignment	**50 marks**
Module Project	**40 marks**
TOTAL	**235 marks**

Remember that Activities 1 to 6 in this Student Module Booklet will prepare you for completing the module project and the module assignments. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Module Review provides a review of the module and an enrichment activity. You may choose to do some or all of the questions in the Module Review. Again, you should compare your answers with the suggested answers provided in the Appendix.



MODULE PROJECT

Lease or Buy a Car?

Beginning the Project

> Your teacher may not require you to complete all the projects provided in this Applie Mathematics 30 course. Contact your teacher and check whether you need to complet the Module 4 project, Lease or Buy a Car?, as part of your assessment.

Are you considering a new vehicle? If you are, there is a lot to think about! Will it be a truck or a car? an import or a domestic model? sporty or practical? What options will it have? Have you considered safety? Does it protect the driver and the passengers? What about engine and body design, fuel economy, environmental safeguards, and repair costs? What do consumer reports have to say?

Will you lease or buy? What will your monthly payments be? What is the warranty and what costs does it cover? How much is insurance? If you lease and decide to purchase t vehicle when the lease expires, how much extra would you have to pay? If you buy, wha will your total cost be? How much will the vehicle depreciate? What will it be worth when you've made your last payment? How easy will it be to sell?

Your Module 4 project, Lease or Buy a Car?, involves examining the relative merits of leasing or buying a vehicle. To assist you in your investigation, you will use the spreadsheet *LEASEBUY*, which is provided on the Applied Mathematics 30 CD.

Turn to page 150 of the textbook and read the last paragraph of "Major Decisions: Lease or Buy a Car? Rent or Buy a House?" Answer the questions posed, and store your responses in the project section of your mathematics binder.

Begin researching the advantages of leasing or buying a vehicle by visiting the website given on page 151 of the textbook. This website has links to several sites you may find helpful.

As you work through Activities 1 through 6, continue to research ideas for leasing or buying a car by surfing the Internet and visiting the websites of car manufacturers and consumer protection agencies, by reading newspaper and magazine advertisements, or by talking to salespeople who work at new- and used-car dealerships.

You will be given more direction on how to complete this project later in this module. In the meantime, feel free to discuss your project with your study partner or a family member. Remember, the work on the project you submit must be your own.

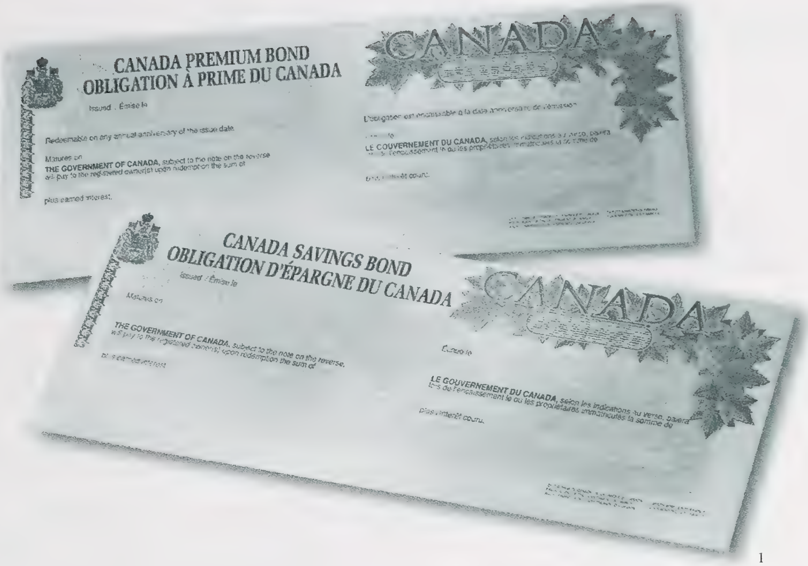
ACTIVITY 1

Investing Money

Matthew has been working for the past year while living at home. As a result, he has managed to save $2500. Because Matthew plans to attend a university or community college in five years, he decided to invest the money in a five-year guaranteed investme certificate (GIC).

A GIC is a conservative investment vehicle that normally bears higher interest than mo savings accounts. The financial institution that issues the certificate guarantees a fixed interest rate for a specified period of time. For example, if the certificate states you will receive 5% per year over five years, that is precisely what you will receive. With a GIC though, you cannot cash it in before the end of the term without penalty. **Note:** There a GICs that can be cashed without penalty at anytime, but the interest rate is usually lowe

Another investment option Matthew could have considered is the purchase of Canada Savings Bonds. Like GICs, Canada Savings Bonds are purchased for a set term, but the are cashable anytime with interest earned up to but not including the month in which th are redeemed. The government of Canada guarantees a minumum interest rate. Howeve if market conditions warrant, the government may adjust the rate upward during the ter of the bonds so holders of the bonds do not cash them in before maturity.



CANADA PREMIUM BOND
OBLIGATION À PRIME DU CANADA

Issued / Émise le

Redeemable on any annual anniversary of the issue date.

Matures on

THE GOVERNMENT OF CANADA, subject to the note on the reverse will pay to the registered owner(s) upon redemption the sum of

plus earned interest.

CANADA

LE GOUVERNEMENT DU CANADA

CANADA SAVINGS BOND
OBLIGATION D'ÉPARGNE DU CANADA

Issued / Émise le

Matures on

THE GOVERNMENT OF CANADA, subject to the note on the reverse, will pay to the registered owner(s) upon redemption the sum of

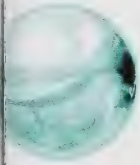
CANADA

LE GOUVERNEMENT DU CANADA, selon les indications au verso, paiera lors de l'encaissement le ou les propriétaires immatriculés la somme de

plus intérêt couru.

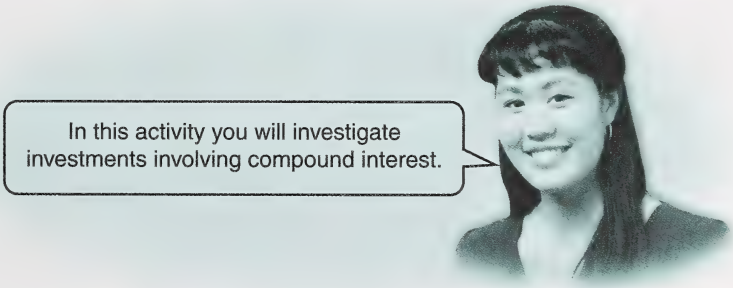
[1]

Purchasers of Canada Savings Bonds can choose between bonds that bear simple interest or compound interest. Simple interest is paid annually on the anniversary of the bond. Compound interest is paid when the bond is redeemed. Like simple interest, compound interest is calculated at regular intervals; however, the compound interest calculated for an interest period is added to the principal for the next interest period.



If you have access to the Internet, you can obtain additional information concerning Canada Savings Bonds at the following website:

http://www.cis-pec.gc.ca



In this activity you will investigate investments involving compound interest.

---

[1] Reproduced with the permission of the Minister of Finance and the Minister of Public Works and Goverment Services Canada, 2001.

Example

Chantal invests \$2500 at an annual interest rate of 6%. If the interest is compounded annually, how much will Chantal have after 5 years?

Solution

When the interest is added at the end of the first year, Chantal's investment will be worth 6% more than it was originally. Because \$2500 is 100% of her original investment, Chantal's investment will be worth 106% when 6% interest is added.

$$\begin{aligned} 106\% \text{ of } \$2500 &= 1.06 \times \$2500 \\ &= \$2650 \end{aligned}$$

When the interest is calculated at the end of the second year, it will be determined using \$2650.

At the end of the second year, Chantal will have 106% of \$2650.

$$\begin{aligned} 106\% \text{ of } \$2650 &= 1.06 \times \$2650 \\ &= 1.06 \times (1.06 \times 2500) \quad \longleftarrow 2650 = 1.06 \times 2500 \\ &= \$2500(1.06)^2 \\ &= \$2809 \end{aligned}$$

Do you see a pattern? For every year the interest is calculated, you multiply by another factor of 1.06.

Therefore, Chantal will have $\$2500(1.06)^5 = \$3345.56$ after 5 years.

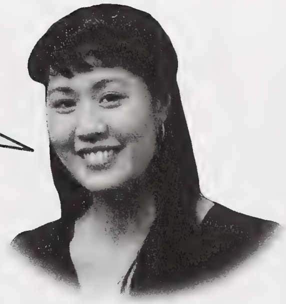
In the previous example, the answer could have been determined by calculating $\$2500(1+0.06)^5$, where \$2500 is the original principal (P), 0.06 (or 6%) is the interest rate per compounding period (i), and 5 is the number of compounding periods (n).

In general, the amount, A, of an investment, where the original principal, P, is bearing an interest rate, i, that is compounded annually for n years, can be determined using the following formula:

$$\boldsymbol{A = P(1+i)^n}$$

Turn to page 1 of Assignment Booklet 4A
and answer question 1.

If the interest rate is calculated more often than once a year, you must modify the formula $A = P(1+i)^n$.



Example

Derek invests \$2500 in an account bearing an annual interest rate of 5%, compounded semi-annually. What is his investment worth after 6 years?

Solution

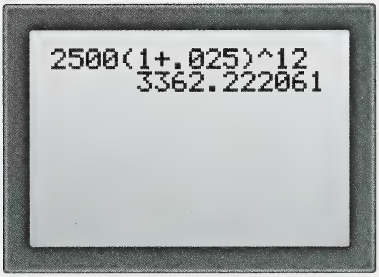
Semi-annually means "twice a year." You must divide the annual rate by 2 to determine the interest Derek will receive every 6 months.

$$\begin{aligned} i &= \frac{5\%}{2} \\ &= 2.5\% \text{ or } 0.025 \end{aligned}$$

Over 6 years the interest will be calculated $2 \times 6 = 12$ times; therefore, $n = 12$.

$$\begin{aligned} \therefore\ A &= P(1+i)^n \\ &= 2500(1+0.025)^{12} \\ &= 3362.22 \end{aligned}$$

Derek's investment will grow to \$3362.22 after 6 years.



```
2500(1+.025)^12
      3362.222061
```

Turn to page 152 of the textbook and read the introductory paragraphs of Tutorial 4.1, "Investing Money."

1. Answer exercises 1, 2, and 3 of "Practise Your Prior Skills" on pages 152 and 153 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 61–62.

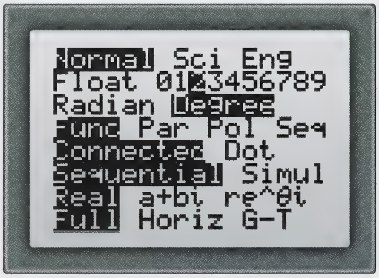
Problems involving compound interest can also be solved using one of the financial features of your graphing calculator. You will use the TVM Solver. *TVM* means "Time Value Money."

Example

Celeste invests $2500 in an account bearing an annual interest rate of 5%, compounded semi-annually. Use your graphing calculator to determine what her investment will be worth after 6 years.

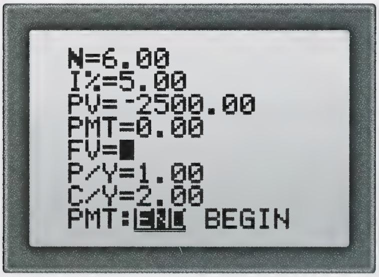
Solution

To get your calculator to round answers to the nearest cent, press MODE and select “2” for the number of decimal places in the answer.



```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T
```

To access the TVM Solver, press 2nd [FINANCE] 1 (1:TVM Solver…). Then enter the values shown on the following display.



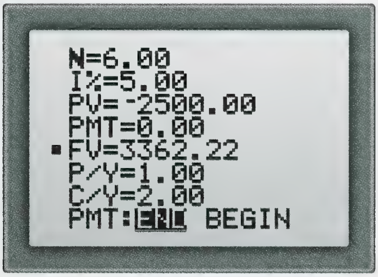
```
N=6.00
I%=5.00
PV=-2500.00
PMT=0.00
FV=■
P/Y=1.00
C/Y=2.00
PMT:END BEGIN
```

Each line in the display represents the following:

- N is the number of years the principal is invested (6 years).
- I% is the annual interest rate (5%).

- PV is the present value or principal invested ($2500). Because the principal is deposited into the account and not an amount received by the person making the investment, it is entered as a negative value. Payments made are negative; amounts received are positive.
- PMT refers to any additional payments made during the term of the investment. In this case, there are no payments.
- FV refers to the future value of the investment. This is what you want to determine in this problem. For the time being, do not enter anything here; it will be calculated later.
- P/Y is the number of deposits or payments per year. Only one payment was made, the original deposit.
- C/Y is the number of times the interest is calculated per year. In this problem the interest is calculated semi-annually (twice a year).

Use the arrow keys to position the cursor on the FV line. This will tell the calculator that it is the FV value that must be calculated. Then press ALPHA [SOLVE].



```
N=6.00
I%=5.00
PV=-2500.00
PMT=0.00
▪FV=3362.22
P/Y=1.00
C/Y=2.00
PMT:END BEGIN
```

Celeste will have $3362.22 in the account after 6 years.

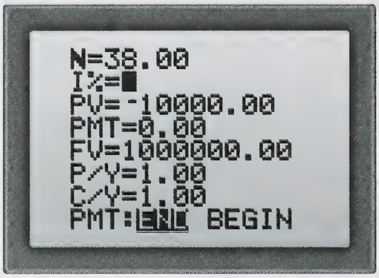
If you know the future value (FV) of an investment earning compound interest, you can use the TVM Solver to determine the principal (PV), annual interest rate (I%), or the number of years (N) the investment is held.

Example

Bahari received a gift of $10 000 from his aunt living in Swift Current. Bahari is 17 years of age and intends to invest the money until he retires 38 years from now. What annual interest rate, compounded annually, must he earn for this sum of money to grow to $1 000 000?

Solution

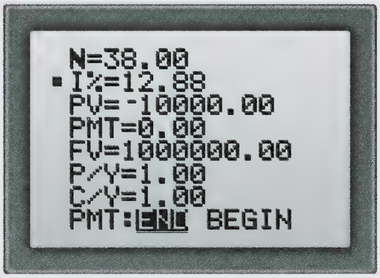
Access the TVM Solver by pressing 2nd [FINANCE] 1 (1:TVM Solver…). Then enter the values shown in the following display.



```
N=38.00
I%=█
PV=-10000.00
PMT=0.00
FV=1000000.00
P/Y=1.00
C/Y=1.00
PMT:END BEGIN
```

← The term is for 38 years.
← The original deposit is $10 000.
← There are no payments made.
← The investment is worth $1 000 000 after 38 years.
← Only the original deposit was made.
← Interest is compounded annually.

Place the cursor beside "I%," and press ALPHA [SOLVE].

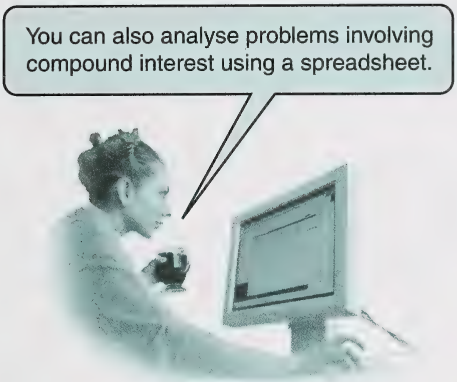


```
 N=38.00
▪I%=12.88
 PV=-10000.00
 PMT=0.00
 FV=1000000.00
 P/Y=1.00
 C/Y=1.00
 PMT:END BEGIN
```

Bahari's investment must earn at least 12.88%, compounded annually, to grow to $1 000 000 in 38 years.

2. Turn to page 153 of the textbook and answer exercise 4 of "Practise Your Prior Skills."

Compare your response with the suggested answer in the Appendix, Activity 1, page 63.


You can also analyse problems involving compound interest using a spreadsheet.

Recall that the explanations in the textbook and in this Student Module Booklet use Microsoft® *Excel*. If you are using a different spreadsheet program, consult the User's Guide.

3. Turn to pages 153 and 154 of the textbook and complete exercises 1 to 5 of "Investigation 1: Watching Money Grow." **Note:** Save the spreadsheet you create, and name it "Investing a Fixed Sum of Money." You will need to use this spreadsheet in Assignment Booklet 4A.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 63–65.

Turn to page 2 of Assignment Booklet 4A and answer question 2.

In the previous investigation you saved your spreadsheet. You can use this spreadsheet as a **template** to analyse problems involving compound interest. A template is a pattern you can use to set up spreadsheets for similar problem situations.

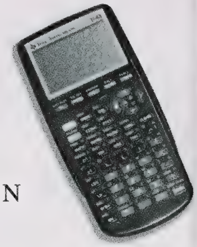
To see an example describing where financial spreadsheets are used in business, view the segment *Spreadsheet Templates* on the Applied Mathematics 30 CD.

In the next investigation you will create a spreadsheet to analyse compound-interest problems involving regular deposits into interest-bearing accounts. Again, you will need to save this spreadsheet so you can use it as a template to analyse similar problems.

4. Turn to pages 155 and 156 of the textbook and complete exercises 1 to 6 of "Investigation 2: Investing Regularly." **Note:** Save the spreadsheet you create, and name it "Investing Regularly." You will need to use this spreadsheet in Assignment Booklet 4A.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 66–68.

In the previous investigation, you used spreadsheets to solve compound interest problems involving regular deposits. You can also solve these problems using the TVM Solver on your graphing calculator. Recall when you used this feature to solve compound interest problems involving a single deposit that the value of N was the number of years the investment bore interest. In the next example, for situations involving regular deposits, you will see that the value of N is the total number of deposits made into the account.



Turn to pages 156 and 157 of the textbook and work through “Example: Investing Frequently.”

5. Answer the following on pages 157 to 159 of the textbook.

   a. exercises 1, 2, and 3 of “Disucssing the Ideas”

   b. exercises 1, 2, 3, 5, 6, and 7 of “Exercises: Checking Your Skills”
   Note: The spreadsheet you create in exercise 7 will be needed in Assignment Booklet 4A.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 68–76.

Turn to page 3 of Assignment Booklet 4A and answer question 3.

Looking Back

In this activity, you investigated compound-interest problems. Some problems involved a single deposit, others involved regular deposits during the time the investment was held. Throughout this activity, you used the formula for compound interest $\left(A = P(1+i)^n\right)$, spreadsheets, and the TVM Solver on your graphing calculator.

6. Turn to page 159 of the textbook and answer “Communicating the Ideas.”

Compare your response with the suggested answer in the Appendix, Activity 1, page 76.

ACTIVITY 2

Investing for the Future

Some people when they first embark on their careers barely give retirement a second thought. Others dream of retiring early with sufficient income to provide them with a comfortable lifestyle that includes travelling to exotic destinations. Certainly, most people out in the cold, shoveling their driveways after a January snow storm, would prefer to be on a tropical beach relaxing and enjoying the sun and scenery.

Most financial advisors encourage their clients not to rely solely on government or company pensions to meet their needs upon retirement. They recommend that people begin to save early for their retirement and take advantage of the power of compound interest.

The federal government, too, encourages people to save for the future. They do this by providing tax advantages to those who participate in a registered retirement saving plan (RRSP). You can lower your taxable income by the amount you contribute to an RRSP. This amount is a percentage of your previous year's income and depends on whether you belong to a retirement plan through your employer. Those who are self-employed or do not have a retirement plan through their employer are allowed to contribute at a higher rate. Did you know interest or income earned within the RRSP is not taxed? You only pay tax on this income when you withdraw money from your RRSP. When you retire, your income will likely be less than when you were working, so the tax rate on your withdrawals should be less.

If you have access to the Internet, you can obtain additional information on RRSPs and determine how quickly your money can grow at the following websites:

- **http://www.rrsp.org/benefits.htm**
- **http://www.altamira.com/toolkit/rrsp+calculator.htm**

In this activity, you will explore the effects of sheltering savings within an RRSP.

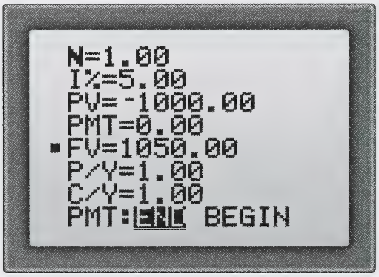
In the following investigation you will set up a spreadsheet to compare tax-sheltered investments in RRSPs to equivalent investments in an unregistered plan.

Turn to page 160 of the textbook and read the introductory paragraphs of Tutorial 4.2, "Investing for the Future."

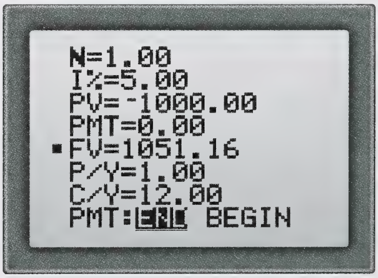
1. Complete exercises 1 to 6 of "Investigation 1: The Power of RRSPs" on pages 160 and 161 of the textbook. The spreadsheet you create will be needed in Assignment Booklet 4A.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 76–78.

The term *simple interest* is applied to interest that applies only to the principle amount. You can see that 5% simple interest for one year gives a different amount than 5% interest compounded monthly for a year (as shown in the following calculator screens).



```
N=1.00
I%=5.00
PV=-1000.00
PMT=0.00
▪FV=1050.00
P/Y=1.00
C/Y=1.00
PMT:END BEGIN
```



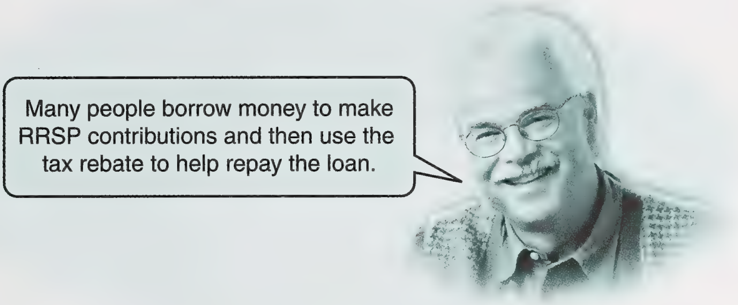
```
N=1.00
I%=5.00
PV=-1000.00
PMT=0.00
▪FV=1051.16
P/Y=1.00
C/Y=12.00
PMT:END BEGIN
```

To allow discussion of simple interest spanning several years, the term *average annual simple rate of return* is used. You can use the following formula to calculate the average annual simple rate of return.

$$\text{Average annual simple rate of return} = \frac{\text{Net return}}{\text{Initial investment} \times \text{Number of years}} \times 100\%$$

2. Determine the average annual simple rate of return for both investments in "Investigation 1: The Power of RRSPs" on pages 160 and 161 of the textbook.

Compare your response with the suggested answer in the Appendix, Activity 2, page 78.


Many people borrow money to make RRSP contributions and then use the tax rebate to help repay the loan.

Turn to pages 4 and 5 of Assignment Booklet 4A and answer question 4.

If you filed an income tax return last year, you would have received from the Canada Customs and Revenue Agency a notice of assessment that includes your RRSP deduction limit for the current taxation year. The total amount you can contribute to an RRSP includes this amount plus any unused amounts from previous years.

If your income is low while you are pursuing your studies and you do not need to contribute to an RRSP to zero out your taxable income, some financial planners recommend you postpone making your contributions to a year when your income is high so you can reduce your tax payable. Others suggest that if you can expect to make a good return on your investments within the RRSP, it is best to contribute as early as you can to take advantage of the power of compound interest. As circumstances vary among individuals, your decision will have to reflect your investment goals; it never hurts to seek expert advice before making your decision.

3. Turn to page 163 of the textbook and answer exercises 2, 3, and 4 of "Discussing the Ideas."

Compare your responses with the suggested answers in the Appendix, Activity 2, page 79.

Example

Gurmehar made a $3000 investment 10 years ago. He has carefully followed the results through the years. He summarized the value of his investment in the folowing table.

Year End	Value
1	$3195.25
2	$3417.63
3	$3636.24
4	$3875.02
5	$4117.10
6	$4371.38
7	$4650.25
8	$4961.90
9	$5283.77
10	$5634.52

a. Perform an exponential regression on this data. What is the regression equation?

b. Predict the value of the investment after 15 years and after 25 years.

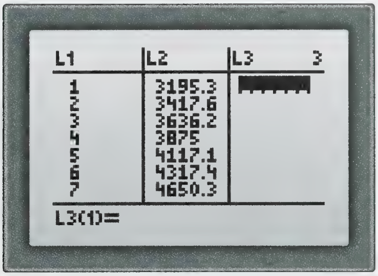
c. What is the approximate annual rate of return?

d. How accurate will the values in the regression equation be to the actual values?

Solution

a. Before you enter any data, press MODE and select "Float" on the second line of the display. Your calculator will no longer display calculations to 2 decimal places.

Step 1: Press STAT 1 (1:Edit…), and enter the data into lists L1 and L2.



L1	L2	L3 3
1	3195.3	▬
2	3417.6	
3	3636.2	
4	3875	
5	4117.1	
6	4317.4	
7	4650.3	

L3(1)=

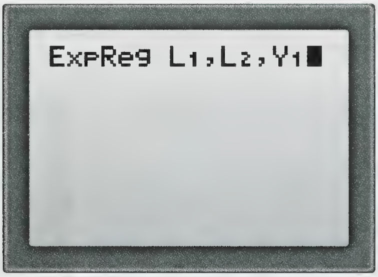
Step 2: Determine the exponential regression equation.

Select the CALC menu.

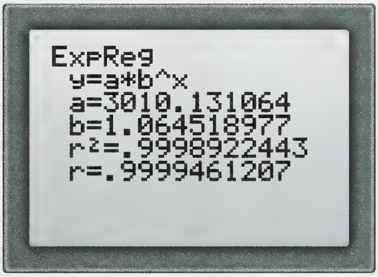
STAT ▶ 0 (0:ExpReg) 2nd [L1] , 2nd [L2] ,

VARS ▶ 1 (1:Function…) 1 (1:Y_1) ENTER

Select the Y-VARS menu.



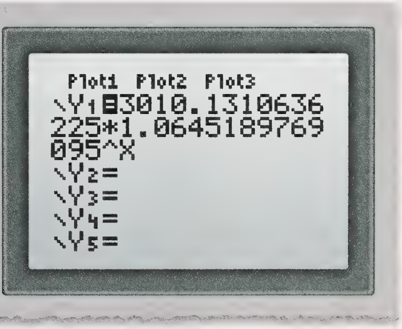
```
ExpReg L1,L2,Y1█
```



```
ExpReg
 y=a*b^x
 a=3010.131064
 b=1.064518977
 r²=.9998922443
 r=.9999461207
```

The exponential regression equation is $y \doteq 3010.13 \times 1.0645^x$.

Note: The "Y_1" part of the Exponential Regression function places the regression equation in the equation editor where specified (in this case Y_1) so you can graph the curve. You can check this by pressing Y=.

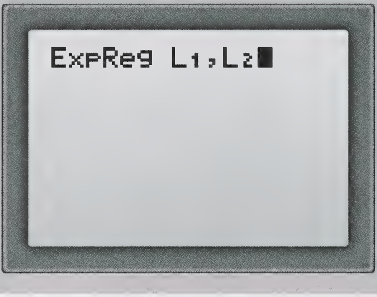


```
 Plot1 Plot2 Plot3
\Y1█3010.1310636
225*1.0645189769
095^X
\Y2=
\Y3=
\Y4=
\Y5=
```

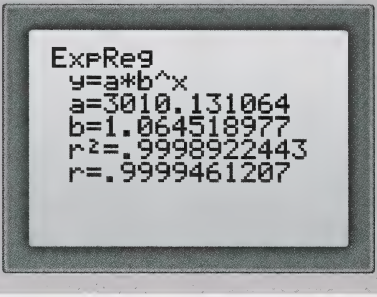
If you only wish to determine the exponential regression equation and not graph the curve, you only need to press the following.

Select the CALC menu.

STAT ▶ 0 (0:ExpReg) 2nd [L1] , 2nd [L2] ENTER

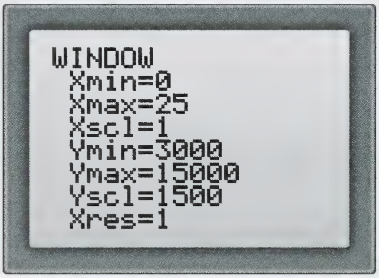


```
ExpReg L1,L2
```

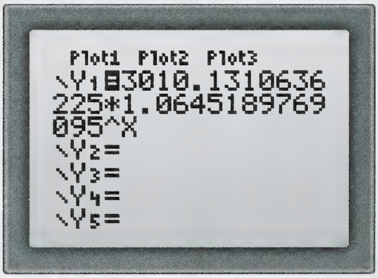


```
ExpReg
 y=a*b^x
 a=3010.131064
 b=1.064518977
 r²=.9998922443
 r=.9999461207
```

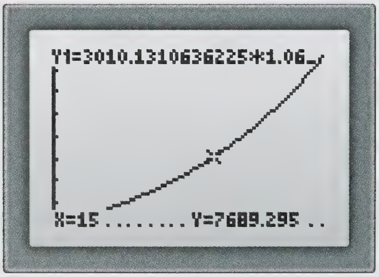
b. The value after 15 years and after 25 years can be calculated by graphing the regression equation and finding the value of y when $x = 15$ and $x = 25$. (The regression equation should already be displayed in the equation editor.) Press GRAPH; then once the graph appears, press 2nd [CALC] 1 (1:value) and enter the desired values ($x = 15$ and $x = 25$).



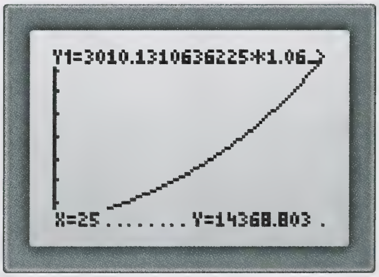
```
WINDOW
 Xmin=0
 Xmax=25
 Xscl=1
 Ymin=3000
 Ymax=15000
 Yscl=1500
 Xres=1
```



```
 Plot1 Plot2 Plot3
\Y1=3010.1310636
225*1.0645189769
095^X
\Y2=
\Y3=
\Y4=
\Y5=
```



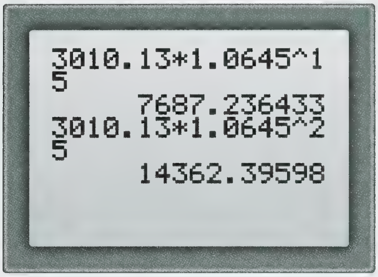
```
Y1=3010.1310636225*1.06
X=15        Y=7689.295
```



```
Y1=3010.1310636225*1.06
X=25        Y=14368.803
```

This method suggests that the value will be about $7689.30 after 15 years and about $14 368.80 after 25 years.

The value after 15 years and after 25 years can also be calculated by substituting $x = 15$ and $x = 25$ into the regression equation.



```
3010.13*1.0645^1
5
        7687.236433
3010.13*1.0645^2
5
        14362.39598
```

This method suggests that the value of the investment will be about $7687.24 after 15 years and about $14 362.40 after 25 years.

c. Notice how the regression equation, $y = ab^x$, parallels the formula for compound interest, $A = P(1+r)^t$.

$$\begin{aligned} y &\doteq 3010.13 \times 1.0645^x \\ &\doteq 3010.13 \times (1 + 0.0645)^x \end{aligned}$$

Therefore, the approximate annual rate of return is 0.0645, or 6.45%.

d. The rate of return in each year is different (as shown in the following table). This makes estimating a future value accurately very difficult.

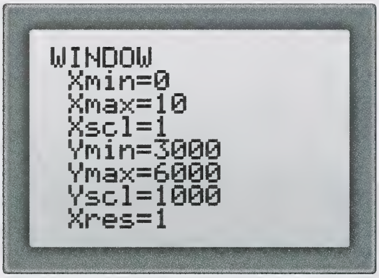
Year	[illegible]	Rate of Return
1	$3195.25	6.51%
2	$3417.63	6.96%
3	$3636.24	6.40%
4	$3875.02	6.57%
5	$4117.10	6.25%
6	$4371.38	6.18%

7	$4650.25	6.38%
8	$4961.90	6.70%
9	$5283.77	6.49%
10	$5634.52	6.64%

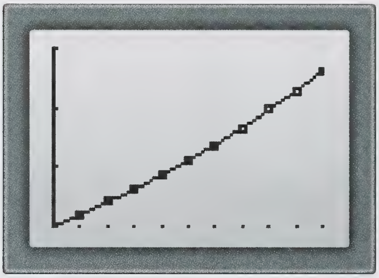
Year 1

$$\text{Rate} = \frac{\$3195.25 - \$3000.00}{\$3000.00} \times 100\%$$
$$\doteq 6.51\%$$

Plotting the given data points with the regression equation shows that the given values fall close to the regression curve, but not exactly on it.



```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=3000
 Ymax=6000
 Yscl=1000
 Xres=1
```



If there are no major changes in the economy, the value of the investment after 15 years and after 25 years will likely be close to the curve. However, the economy generally goes through dramatic changes over a 15-year period. The following chart shows some financial information for a period in the recent past, which indicates a considerable variability in the North American economy.

Year	Canadian Mortgage Rate	American Inflation Rate
1979	12.0%	11.3%
1980	14.5%	13.5%
1981	18.4%	10.4%
1982	18.0%	6.2%

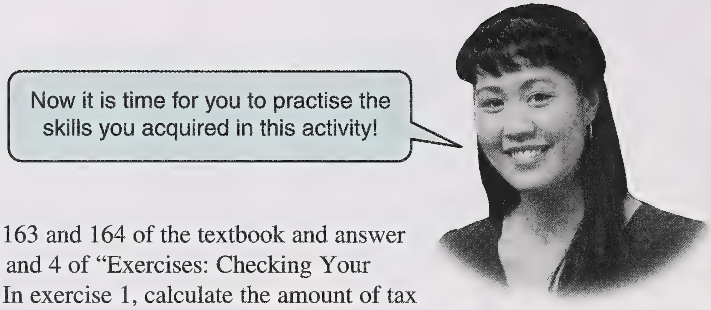
1983	13.6%	3.2%
1984	13.6%	4.4%
1985	12.1%	3.5%
1986	11.2%	1.9%
1987	11.2%	3.7%
1988	11.7%	4.1%
1989	12.1%	4.8%
1990	13.4%	5.4%
1991	11.1%	4.2%
1992	9.5%	3.0%
1993	8.9%	3.0%
1994	9.4%	2.6%
1995	8.1%	2.8%
1996	7.1%	3.0%

In Investigation 1 you explored how a single contribution to an RRSP can grow much more quickly than the same dollar investment outside an RRSP. In the next investigation you will explore how regular contributions can assist you in accumulating wealth. You will use both a spreadsheet and your graphing calculator in your investigation.

4. Turn to pages 161 and 162 of the textbook and complete exercises 1 to 8 of "Investigation 2: So, You Want to Be a Millionaire!"

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 80–82.

Turn to page 6 of Assignment Booklet 4A and answer question 5.


Now it is time for you to practise the skills you acquired in this activity!

5. Turn to pages 163 and 164 of the textbook and answer exercises 1, 3, and 4 of "Exercises: Checking Your Skills." **Note:** In exercise 1, calculate the amount of tax paid on each closing balance at a rate of 44%.

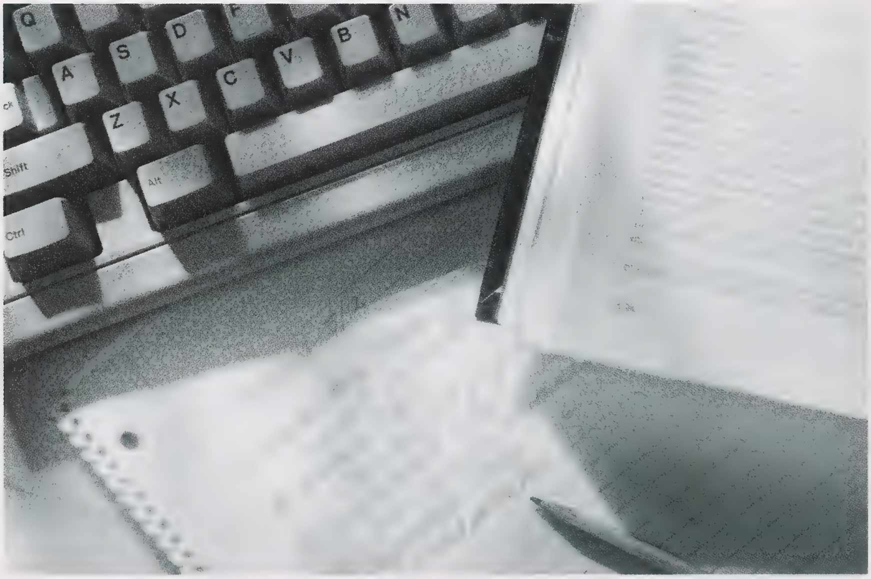
Compare your responses with the suggested answers in the Appendix, Activity 2, pages 82–85.

Looking Back

In this activity, you investigated the effects of sheltering savings by investing in RRSPs, determined how to accumulate wealth through regular savings, and discussed the importance of investing early in life to take advantage of compounding interest.

6. Turn to page 165 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 2, page 86.



ACTIVITY 3

Investment Portfolios

Do you or members of your family own **stock**, or **shares**, in a publicly traded company? When you or other members of the public buy stock in a company, you purchase a share in the assets, or **equity**, of the company. You actually own a piece of the company!

Companies issue stocks to raise capital. The capital they raise can be used for a variety of purposes: start-up costs, research and development, expenditure on buildings and equipment, and business expansion to name a few.

Shares are bought and sold through the stock market. The values of shares rise and fall with the fortunes of the companies that issued them, with the optimism or pessimism of investors, and with the state of national and international economies.

Many companies issue dividends on their shares, which are like interest payments paid to their shareholders. Dividends are just one way shareholders can profit. If the stock price rises, the investor can sell his or her shares for a **capital gain**. However, if the stock price falls and the investor sells, he or she suffers a **capital loss**.

Because the potential exists for losing money, stocks are considered to be a riskier investment than term deposits, GICs, or bonds. In this activity you will explore investment **portfolios** and the proportion of investments within these portfolios that are risk-free and the proportion that involve risk.

Turn to pages 166 and 167 of the textbook and read the introductory paragraphs of Tutorial 4.3, “Investment Portfolios.”

1. Answer exercises 1 and 2 of “Discussing the Ideas” on page 170 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 3, page 86.

If you study the table on page 167 of the textbook, you will notice that financial advisors recommend that the investment mix in your portfolio should reflect your personal circumstances.

When you get your first full-time job, your income will likely be lower than it will be later in your career. In spite of your limited income, your expenses can be high at this stage of your life. If you purchase a vehicle, a home, or even furniture, you will have payments to make. If you marry or start a family, your financial responsibilities increase. For these reasons, you should select a larger proportion of safer investments, such as bonds and GICs, and a smaller proportion of riskier investments in equities, such as stocks and mutual funds. You want to preserve capital so you can meet emerging expenses. This investment phase generally lasts until your mid-thirties.

From your mid-thirties to age fifty, you should be established in your career and have more disposable income. Your investment goals can be longer term and emphasize growth. Your portfolio should be more risk-oriented. Investment advisors recommend you increase the proportion of equities to realize higher returns. You can afford to weather fluctuations in the markets without the necessity to cash in your investments at a loss. With more options than the more conservative investments of bonds and GICs, you can look towards higher yields over the long term.

In your fifties and early sixties, you are preparing for retirement. You will probably reduce the risk in your portfolio. Preserving capital is important because you will have less time to recover from a bear (declining) market or market correction. The investment mix in your portfolio will be closer to what it was when you first started investing.

Of course, in retirement, your portfolio should emphasize safety, with a much higher proportion of safe investments than stocks or mutuals.

In the next few examples, you will explore investment options and how to determine the return on your investments.

Turn to pages 167 to 170 of the textbook and work through “Example 1: Allocation of Resources,” “Example 2: Rates of Return,” and “Example 3: Analyze an Investment Portfolio.”

In Examples 2 and 3 you actually calculated the average annual **compound** rate of return. This rate gives the average compound rate of an investment over a period of time and is considerably lower than the average annual simple rate of return as time increases. Throughout this course, the average annual compound rate of return is simply referred to as the average annual rate of return.

2. Answer exercises 2, 4, 5, and 6 of “Exercises: Checking Your Skills” on pages 171 to 172 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 87–91.

Turn to pages 7 and 8 of Assignment Booklet 4A and answer questions 6,7, and 8.

Looking Back

In this activity, you examined investment portfolios and investment options and determined how to calculate returns on investments. To examine an actual investment portfolio and the underlying investment decisions, view the segments *Investments: An Institutional Portfolio, Investments: An Individual Portfolio Part 1*, and *Investments: An Individual Portfolio Part 2* on the Applied Mathematics 30 CD.

3. Turn to page 173 of the textbook and answer “Communicating the Ideas.”

Compare your response with the suggested answer in the Appendix, Activity 3, page 92.

ACTIVITY 4

Leasing

Alexa would like a new car. She has decided on the make and model, but she hasn't decided whether to lease or buy.

Alexa knows that if she leases the vehicle for three years, her monthly payments will be less than her monthly payments if she were to borrow the money to buy the car over three years. Her friends have even suggested that she could take advantage of this difference and drive a more expensive car.

A salesperson at the car dealership told her that if she leases the car for three years, she can simply return the car at the end of the lease and walk away or lease another brand new vehicle. This will save her the hassle of selling the car or trading it in if she outright purchased it. The salesperson also reassured her that if she wants to purchase the car at the end of her lease, she has the option of paying the purchase price that appears in the lease agreement.

This guaranteed purchase price at the end of the lease is known as the **residual value** of the vehicle. Alexa was told that the lease payments are really loan payments and that the "loan" she would be repaying if she leased the car is the difference between the selling price less her down payment and the residual value. So, if Alexa wants to reduce her lease payments she can increase her down payment.

What do you think Alexa should do?

In this activity you will analyse the relative merits of buying and leasing. You will use a spreadsheet to assist you in your analysis; but, first, you will examine the residual value of a vehicle at the end of a lease.

Turn to page 174 of the textbook and read the introductory paragraphs of Tutorial 4.4, "Leasing."

1. Complete exercises 1, 2, and 3 of "Investigation 1: Leases" on pages 174 and 175 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 92–93.

Example

Natsu bought a new car for $29 900. He has carefully followed the value of his car through its first 5 years. The value of his car is summarized in the following table.

Year	Value
0	$29 900
1	$20 000
2	$17 000
3	$15 250
4	$12 250
5	$10 500

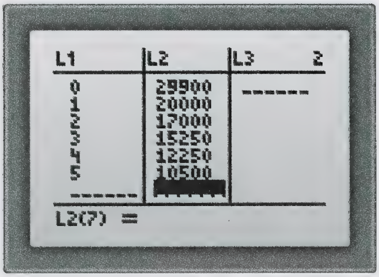
a. Determine the exponential regression equation for this data.

b. Predict the value of Natsu's car after 10 years and after 14 years.

c. What is the approximate annual rate of depreciation?

Solution

a. **Step 1:** Enter the data into lists L1 and L2.



```
L1      L2      L3     2
  0      29900  ------
  1      20000
  2      17000
  3      15250
  4      12250
  5      10500
------
L2(7) =
```

Step 2: Determine the exponential regression equation.

Select the CALC menu.

STAT ▶ 0 (0:ExpReg)

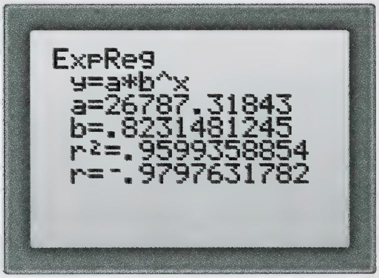
2nd [L1] , 2nd [L2]

Select the Y-VARS menu.

, VARS ▶ 1

(1:Function…) 1 (1:Y_1)

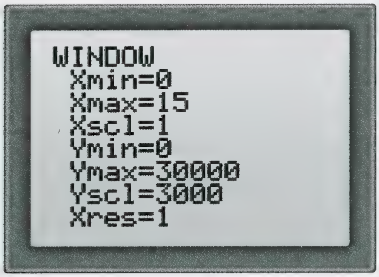
ENTER



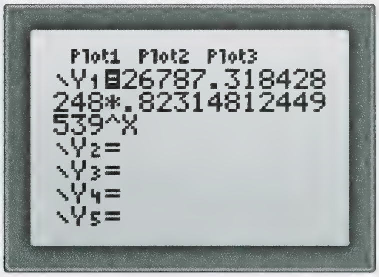
```
ExpReg
 y=a*b^x
 a=26787.31843
 b=.8231481245
 r²=.9599358854
 r=-.9797631782
```

The exponential regression equation is $y \doteq 26\,787.32 \times 0.8231^x$.

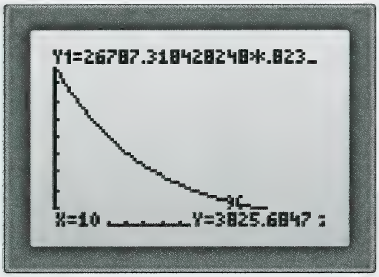
b. The value after 10 years and after 14 years can be calculated by graphing the regression equation and finding the value of y when $x = 10$ and $x = 14$. **Remember:** By including "Y_1" in the Exponential Regression function, the equation is already displayed in the equation editor. So, simply press GRAPH. Once the graph is displayed, press 2nd [CALC] 1 (1:value) and enter the desired values ($x = 10$ and $x = 14$).



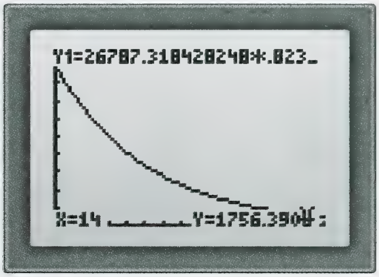
```
WINDOW
 Xmin=0
 Xmax=15
 Xscl=1
 Ymin=0
 Ymax=30000
 Yscl=3000
 Xres=1
```



```
 Plot1 Plot2 Plot3
\Y1■26787.318428
248*.82314812449
539^X
\Y2=
\Y3=
\Y4=
\Y5=
```



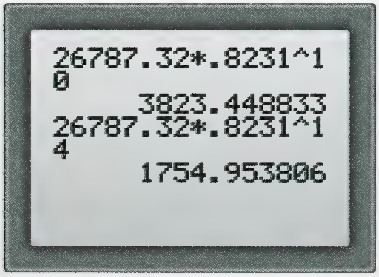
```
Y1=26787.318428248*.823_
X=10          Y=3825.6847
```



```
Y1=26787.318428248*.823_
X=14          Y=1756.390
```

This method suggests that the value of Natsu's car will be about \$3825.68 after 10 years and about \$1756.39 after 14 years.

The value after 10 years and after 14 years can also be calculated by substituting $x = 10$ and $x = 14$ into the regression equation.



```
26787.32*.8231^1
0
        3823.448833
26787.32*.8231^1
4
        1754.953806
```

This method suggests that the value of Natsu's car will be about \$3823.45 after 10 years and about \$1754.95 after 14 years.

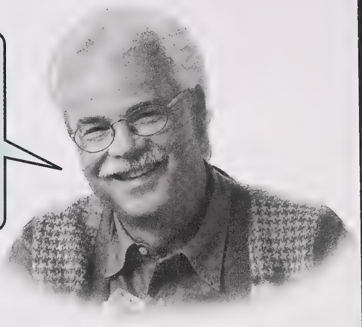
c. $y \doteq 26\,787.32(0.8231)^x$

$\doteq 26\,787.32\left[1+(0.8231-1)\right]^x \quad \longleftarrow A = P(1+i)^n$

$\doteq 26\,787.32\left[1+(-0.1769)\right]^x$

The approximate annual rate of depreciation is 17.69%.

In the next investigation, you will use a spreadsheet to compare the relative merits of purchasing or leasing the same vehicle. From the Applied Mathematics 30 CD, open the spreadsheet *LEASEBUY*. Use this template to complete the following exercises.



2. Turn to pages 175 to 177 of the textbook and complete exercises 1 to 4 of "Investigation 2: To Lease or to Buy?" Even though the *LEASEBUY* template is already set up for you, still read through exercise 1 for a little bit of background on the spreadsheet.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 94–96.

You can use the TVM Solver on your graphing calculator to calculate the residual value of a lease vehicle. Remember, a lease is really a loan. The amount of the loan is the difference between the purchase price (reduced by the down payment) and the residual value. Lease payments are due at the beginning of each month, and interest is also calculated at the beginning of each month.

Example

A car dealership is advertising a lease rate of 1.9% over 48 months. The car that Kaya wants to purchase has a purchase price of $29 900. At the end of the lease, the car can be purchased for $11 000. What would the anticipated monthly lease payments be?

Solution

Use the TVM Solver to find the approximate monthly lease payments.

N is 48, since there are a total of 48 monthly payments.

I% is 1.9%, since the lease rate is 1.9%.

PV (present value) is $29 900, since Kaya would be receiving this amount.

PMT is what you need to determine.

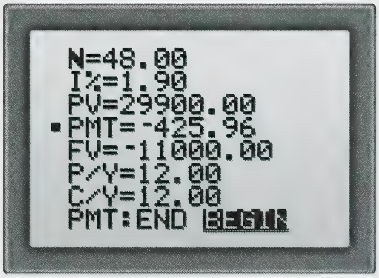
FV (future value) is $11 000, since this is the amount Kaya would be giving back.

P/Y is 12, since the payments are monthly.

C/Y is 12, since interest is compounded monthly.

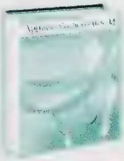
BEGIN is selected, since the payments occur at the beginning of each month.

Now, place the cursor beside "PMT" and press ALPHA [SOLVE].



```
 N=48.00
 I%=1.90
 PV=29900.00
▪PMT=-425.96
 FV=-11000.00
 P/Y=12.00
 C/Y=12.00
 PMT:END BEGIN
```

Kaya would be looking at lease payments of about $426 per month. (**Remember:** A negative value shows that this amount is to be paid.) If Kaya is a good negotiator, the real lease payments could be much less than this.



Turn to pages 177 and 178 of the textbook and work through "Example: Lease-End Value."

3. Answer the following on pages 178 to 180 of the textbook.

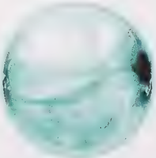
- **a.** exercises 1 to 4 of "Discussing the Ideas"
- **b.** exercises 1, 2, and 7 of "Exercises: Checking Your Skills"
  Note: In exercise 2.c., the 42% residual is for a 3-year lease, not a 2-year lease.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 96–101.

Turn to pages 9 to 12 of Assignment Booklet 4A and answer questions 9 to 12.

Project: Landscape Gardening

Plants and open spaces are a part of our human heritage. The urban landscape that most Canadians live in seems to be more asphalt, concrete, brick, wood, and stone than the living greenery our ancestors grew up in. The explosion of interest in gardening and home landscaping is a testament to the enjoyment of plants that runs through the human subconscious. It also means that designing a small oasis of nature into the urban landscape is becoming a more significant part of modern achitecture and urban planning.



You might find the following websites a useful source of information about gardens, gardening, and landscaping:

- **http://www.hgtv.ca/garden/articles**
- **http://www.icangarden.com**
- **http://www.discoveredmonton.com/devonian**

Turn to pages 68 to 70 of the Project Book and read "The Task" and "Background" of the Landscape Gardening project. Read the questions posed in "Background," and write your responses in your mathematics binder.

You are going to work with the landscaping plans given in the Project Book. You will develop an invoice for the preparation of the small college dormitory garden that is described there. Along the way, you will practise using some volume, area, and costing calculation skills that you have mastered in earlier courses as well as your newly developed spreadsheet skills.

4. **a.** Answer exercise 1 of "Getting Started" on page 70 of the Project Book. Use the following chart to direct your thinking. **Note:** The flower beds run along walls of the building except for the one wall.

Area of Flower Beds	
Landscape Fabric Required	
Total Cost of Fabric	
Mulch Volume Required	
Total Cost of Mulch	

b. Answer exercise 2 of "Getting Started" on page 70 of the Project Book. You may find the following table simplifies the task.

Item	Total Number of Plants	Area (m^2)	Depth (m)	Approximate Volume (m^3)
Lawn	N/A		0.10	
Trees (willows and maples)				
Shrubs	____ Bamboo ____ Cedar ____ Rhododendron			
Boxwood hedges	N/A			
Bedding plants	N/A			
Rock garden	N/A			
			Total	

c. Answer exercise 3 of "Getting Started" on page 70 of the Project Book.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 102–103.

Turn to pages 13 to 17 of Assignment Booklet 4A and answer questions 13 to 17.

Looking Back

In this activity, you explored the relative merits of buying or leasing a vehicle. You will need the skills you acquired and the *LEASEBUY* spreadsheet to complete the module project. You also completed a project that involved costing and then altering a landscape design to fit within a certain budget.

5. Turn to page 181 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 4, page 103.

ACTIVITY 5

Mortgages

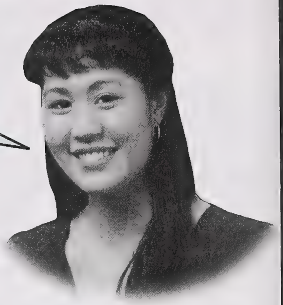
For most Canadians, purchasing a house will be the largest investment decision they will make in their lives. A significant part of this investment decision is the amount of money that must be borrowed to finance the house, what the monthly payments will be, and the number of years it will take to pay off the loan.

Loans taken out to purchase a house (or other forms of real estate) are called **mortgages**. If you take out a mortgage, the lender will have legal claim on the property. This is to protect them if you do not meet your financial obligations. The interest rate on a mortgage is called the **mortgage rate**. Making payments to pay off a mortgage is called **amortization**. The **amortization period** is the number of years required to pay off or amortize the mortgage.

Your mortgage payment will depend on the interest rate, the amount of the mortgage, and the amortization period. Amortization tables can be purchased to determine your monthly payments. If you decide to purchase these tables, make certain they are for Canadian and not American mortgages. In Canada, interest on your mortgage is compounded every six months, whereas interest is compounded monthly in the United States.

If you have access to the Internet, you can access a number of sites that will calculate the monthly payment of your mortgage when you enter the value of the mortgage, the interest rate, and the amortization period. Here are a few websites to consider:

- **http://www.canadamortgage.com**
- **http://www.wblady.com/mortgageCalcApplet.html**
- **http://www.altainfo.com/calculators.htm**



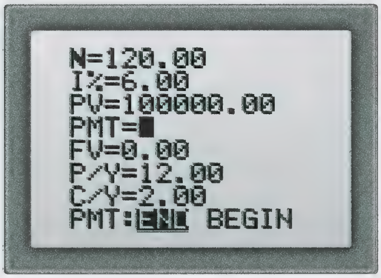
You can also use your graphing calculator to determine monthly mortgage payments.

Example

Use your graphing calculator to determine the monthly payments for a $100 000 mortgage at 6%, amortized over 10 years.

Solution

Activate the TVM Solver by pressing 2nd [FINANCE] 1 (1:TVM Solver…), and enter the values.

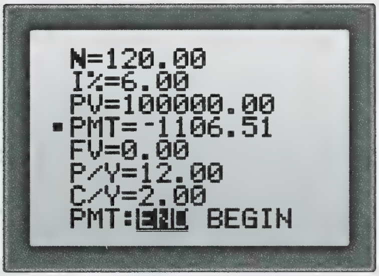


```
N=120.00
I%=6.00
PV=100000.00
PMT=■
FV=0.00
P/Y=12.00
C/Y=2.00
PMT:END BEGIN
```

Remember: When using the TVM Solver on your graphing calculator, have your calculator automatically round answers to 2 decimal places. This will make reading the answer much easier.

There are 120 payments over 10 years, so $N = 120$.
The interest rate is 6%, so $I\% = 6$.
The mortgage is for \$100 000, so $PV = 100\ 000$.
The mortgage is paid out, so $FV = 0$.
There are 12 payments made each year, so $P/Y = 12$.
This is a Canadian mortgage, so $C/Y = 2$.
Payments are made at the end of each period, so select "END."

After you have entered the appropriate values, position the cursor next to PMT and press ALPHA [SOLVE].



```
N=120.00
I%=6.00
PV=100000.00
▪PMT=-1106.51
FV=0.00
P/Y=12.00
C/Y=2.00
PMT:END BEGIN
```

The monthly payment will be $1106.51. **Remember:** Because the payment is money going out, it is negative.

In the following investigation, you will explore additional aspects of Canadian mortgages.

Turn to page 184 of the textbook and read the introductory paragraphs of Tutorial 4.5, "Mortgages."

1. Complete exercises 1 to 5 of "Investigation 1: Which House Can You Afford?" on pages 184 and 185 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 103–104.

Turn to page 1 of Assignment Booklet 4B and answer question 1.

Purchasing a house is sometimes called buying an appreciating asset. This is because houses often grow in value over the years. Here is an example that shows this growth.

Example

The Callihoos purchased a home 10 years ago for \$120 000. They have seen the value of their house increase over the years. The following chart summarizes how the value of their home has changed.

Year	Value
0	\$120 000
1	\$123 400
3	\$130 900
5	\$138 800
7	\$147 400
9	\$155 900
10	\$160 300

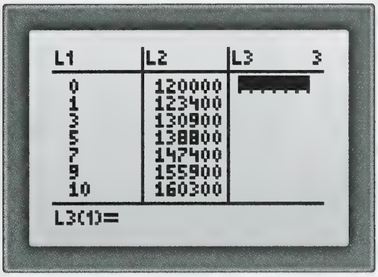
a. Determine the exponential regression equation for this data.

b. Predict the value of the Callihoos' home after 15 years and after 25 years. Round your answer to the nearest hundred.

c. What is the approximate annual rate of return?

Solution

a. Enter the data into lists L1 and L2.



```
L1     L2      L3      3
0      120000
1      123400
3      130900
5      138800
7      147400
9      155900
10     160300
L3(1)=
```

Find the exponential regression equation by pressing the following.

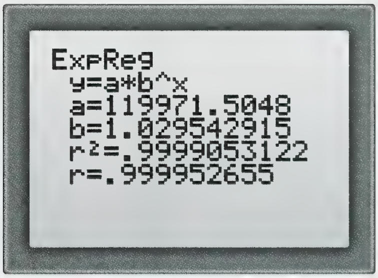
Select the CALC menu.

STAT ▶ 0 (0:ExpReg) 2nd [L1] , 2nd [L2] , VARS

Select the Y-VARS menu.

▶ 1 (1:Function...) 1 (1:Y_1)

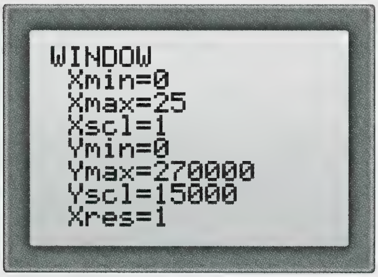
ENTER



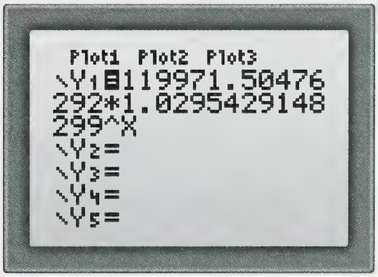
```
ExpReg
 y=a*b^x
 a=119971.5048
 b=1.029542915
 r²=.9999053122
 r=.999952655
```

The exponential regression equation is $y \doteq 119\,971.50 \times 1.0295^x$.

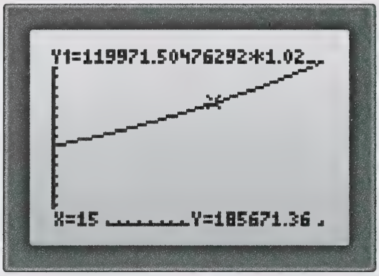
b. The value of the Callihoos' home after 15 years and after 25 years can be calculated by finding the value of y when $x = 15$ and when $x = 25$. (**Remember:** the regression equation should already appear in the calculator's equation editor.)

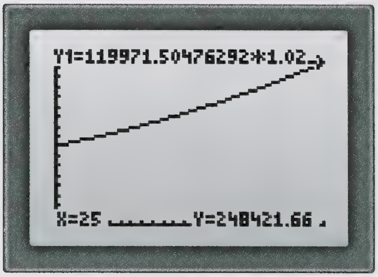


```
WINDOW
 Xmin=0
 Xmax=25
 Xscl=1
 Ymin=0
 Ymax=270000
 Yscl=15000
 Xres=1
```

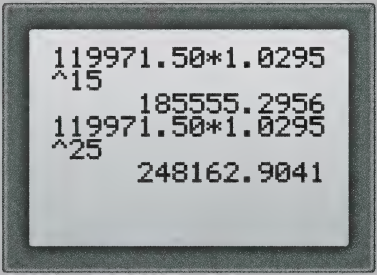


```
 Plot1 Plot2 Plot3
\Y1=119971.50476
292*1.0295429148
299^X
\Y2=
\Y3=
\Y4=
\Y5=
```





The home will be valued at about \$185 700 after 15 years and about \$248 400 after 25 years.

> **Note:** The value after 15 years and after 25 years can also be done by evaluating $119\,971.50 \times 1.0295^{15}$ and $119\,971.50 \times 1.0295^{25}$ on your calculator. This is shown in the following calculator screen.
>
> 
>
> ```
> 119971.50*1.0295
> ^15
>          185555.2956
> 119971.50*1.0295
> ^25
>          248162.9041
> ```
>
> Because this method uses rounded values from the regression equation, the values are not the same as those given by graphing.

c. Notice how similar the regression equation is to the formula for compound interest, $A = P \times (1+r)^t$. The approximate annual rate of appreciation (growth in value) as shown by the regression equation $\left(y \doteq 119\,971.50 \times 1.0295^x\right)$ is $1.0295 - 1 = 0.0295$, or 2.95%.

In the next examples, you will explore how changing the amortization period affects the monthly payment and what the balance of the mortgage will be after you have started making payments.

Turn to pages 186 to 187 of the textbook and work through "Example 1: The Effect of Changing Amortization Periods" and "Example 2: The Outstanding Balance on a Mortgage."

2. Answer the following on pages 187 to 190 of the textbook.

- **a.** exercises 1, 2, and 3 of "Discussing the Ideas"
- **b.** exercises 1.a., 1.b., 2, 3.a., 3.b., 3.c., 5.a., 6, and 7 of "Exercises: Checking Your Skills" (**Note:** In exercise 5, the heading of the third column of the first table should be Total Payments, not Monthly Payments.)

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 104–111.

Turn to pages 2 to 4 of Assignment Booklet 4B and answer questions 2 and 3.

Looking Back

In this activity, you explored mortgage payments. You discovered that if you decrease the amortization period, your monthly payments increase, but the total cost of repaying the mortgage decreases. Using your graphing calculator, you solved a variety of mortgage problems.

3. Turn to page 190 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 5, page 111.

ACTIVITY 6

Renting Versus Buying

Simeon and Masha have managed to save $20 000. They could use the money as a down payment on a new home or invest the $20 000 and rent an apartment. They know that regardless of which decision they make, there will be a variety of expenses associated with buying or renting.

If they buy, they will have mortgage payments and have to pay taxes, utilities, insurance, and basic maintenance (such as shingling the roof and mowing the lawn). If they rent, they will still insure their belongings and pay some utilities (likely only power, telephone and cable). How would you advise them? What additional information would you need? Do you think they should base their decision on monetary considerations alone?

In this activity, you will explore the benefits of home ownership and renting, using a spreadsheet in your exploration. For an introduction to the use of spreadsheets to compare buying and renting, view the segment *Renting Versus Buying a Home* on the Applied Mathematics 30 CD.

Canada Mortgage and Housing Corporation (CMHC) has detailed information on the costs of buying and owning a home. You can access their website at

http://www.cmhc-schl.gc.ca/cmhc.html

Also, some realtors have websites that offer advice on buying and renting. One such site includes a calculator that allows you to input data for both buying and renting to make cost and benefit comparisons. It is as follows:

http://www.canadamortgage.com/calculators/rentvsown.cgi

Before you begin to work on your spreadsheet, you will review determining the property taxes from the assessed value of your home.

Example

In Edmonton, property taxes are based on the current market value. If the Berards' property is assessed at $130 000 and the current year's combined municipal and school tax rate is 15.1888 mills, what are their property taxes?

Solution

Recall that the formula to determine property tax is as follows:

$$\text{Property tax} = \frac{\text{Assessed value} \times \text{Mill rate}}{1000}$$

$$\begin{aligned} \therefore \text{ Berards' property tax} &= \frac{\$130\ 000 \times 15.1888}{1000} \\ &= \$1974.54 \end{aligned}$$

Turn to page 191 of the textbook and read the introductory paragraphs of Tutorial 4.6, "Renting Versus Buying."

1. Answer exercise 1.a. of "Practise Your Prior Skills" on page 191 of the textbook.

Compare your response with the suggested answer in the Appendix, Activity 6, page 111.

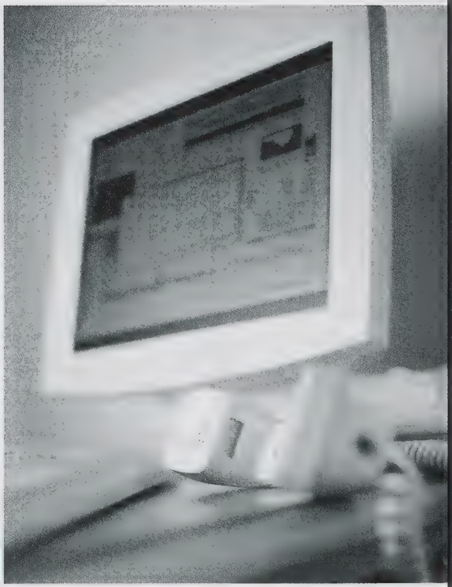
As part of your preparation for the investigation of whether it is better to buy or rent, research the answers to the following exercises on household expenses.

2. Turn to page 194 of the textbook and answer exercises 1 to 4 of "Discussing the Ideas."

Compare your responses with the suggested answers in the Appendix, Activity 6, page 112.

In the following investigation, you will use a spreadsheet to compare buying and renting. Open the spreadsheet *RENTBUY* on the Applied Mathematics 30 CD. This spreadsheet automatically calculates the regular payments of a mortgage, so you do not need your TVM Solver to determine the monthly payments.

3. Turn to pages 192 and 193 of the textbook and complete exercises 1 to 9 of "Investigation: Is It Better to Rent or to Buy?" **Note:** For exercise 6, the amount owing on the house is listed in column K, not column L. Also, the market value in cell P5, not cell Q4, is the purchase price plus the annual increase in the value of real estate.



Compare your responses with the suggested answers in the Appendix, Activity 6, pages 112–117.

You should remember that there are many reasons for deciding whether to buy or rent. Not all of these reasons are financial. Also, the spreadsheet you set up or the websites you may visit can only project the benefits or drawbacks based on best guesses for income security, inflation, increases in utility costs, interest rates, increases or decreases in property values, taxes, and rents. Before making major investment decisions, it is wise to seek expert advice.

4. Turn to pages 194 and 195 of the textbook and answer exercises 1 and 2 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 6, pages 117–119.

Turn to pages 4 to 8 of Assignment Booklet 4B and answer questions 4 to 8.

Looking Back

In this activity, you used a spreadsheet to explore the relative merits of buying or renting.

5. Turn to page 197 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 6, page 120.

Module Review

This module dealt with Chapter 4: Personal Finance in the *Addison-Wesley Applied Mathematics 12 Source Book.*

Turn to page 200 of the textbook and review the skills and concepts that were developed in this module. Also, read the important results and formulas you discovered.

1. Answer exercise 1 of Part A of "What Should I Be Able to Do?" on page 201 of the textbook.

2. Answer exercises 2, 5, 6, 7, and 9 of Part B of "What Should I Be Able to Do?" on pages 201 to 204 of the textbook.

Compare your responses with the suggested answers in the Appendix, Module Review, pages 120–125.

Turn to pages 8 to 20 of Assignment Booklet 4B and complete the Module Review Assignment.

If you had difficulties understanding the skills and concepts in Module 4: Personal Finance, it is recommended that you contact your teacher for some extra help activities. If you have a clear understanding of the skills and concepts in this module, you may wish to do the following enrichment activity. You may decide to do both.

Enrichment

When you are applying for a loan or credit or when you are opening an investment account, the financial institution often requires you to provide information that allows them to calculate your approximate **net worth**. Your net worth is the difference between your **assets** and your **liabilities**; that is, it is the difference between what you own and what you owe.

Net worth = Total assets – Total liabilities

Example

Sven is applying for a loan at his local bank. On the application form, he is asked to list his assets and liabilities. Sven has \$3500 in a savings account, \$750 in a chequing account, \$2000 in bonds, and \$6500 in RRSPs. He also has a late-model car worth \$12 000 if he were to sell it and \$2500 in furniture in his apartment. However, Sven still owes \$10 000 on his car and has a \$15 000 student loan to repay. What is Sven's net worth?

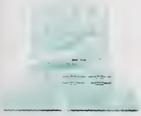
Solution

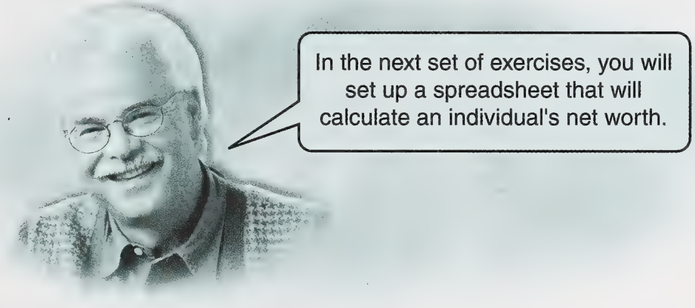
$$\begin{aligned}\text{Total assets} &= \$3500 + \$750 + \$2000 + \$6500 + \$12\,000 + \$2500 \\ &= \$27\,250\end{aligned}$$

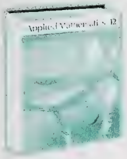
$$\begin{aligned}\text{Total liabilities} &= \$10\,000 + \$15\,000 \\ &= \$25\,000\end{aligned}$$

$$\begin{aligned}\text{Net worth} &= \text{Total assets} - \text{Total liabilities} \\ &= \$27\,250 - \$25\,000 \\ &= \$2250\end{aligned}$$

Sven's net worth is \$2250.



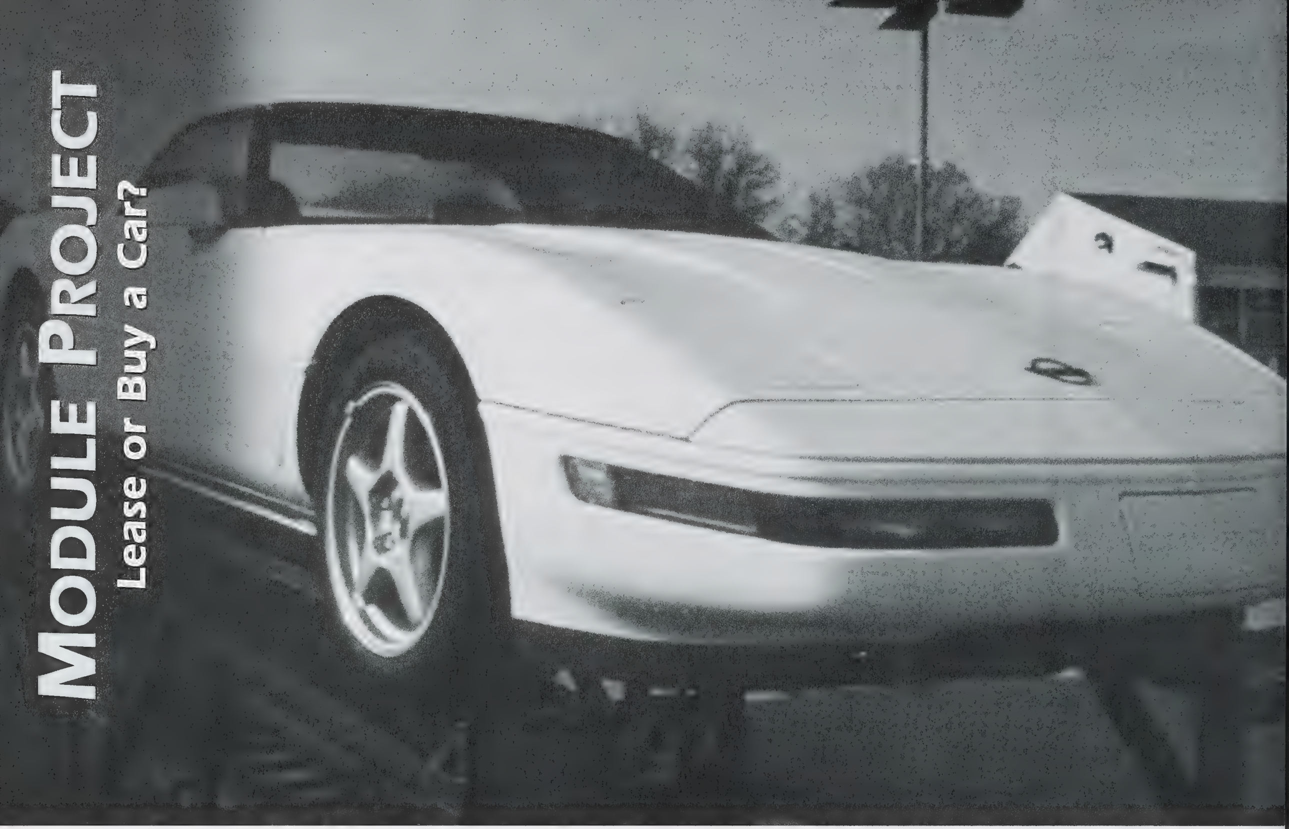

In the next set of exercises, you will set up a spreadsheet that will calculate an individual's net worth.



1. Turn to pages 202 and 203 of the textbook and answer exercises 3 and 4 of Part B of "What Should I Be Able to Do?"

2. Why do you think an outstanding mortgage is subtracted from total liabilities in the formula for the debt/equity ratio?

Compare your responses with the suggested answers in the Appendix, Module Review: Enrichment, pages 125–126.



Completing the Project

By now you should have completed your initial research for the Module 4 project, Lease or Buy a Car? You should have a good understanding of how a lease works—what is meant by the option-to-purchase price (or residual value) at the end of the lease, how the monthly payments are calculated, what additional charges there are, and what taxes you have to pay. You should also have a good understanding of a vehicle purchase plan—how the down payment and interest rates affect your monthly loan payments and how to calculate the total cost of your vehicle.

For your analysis in this project, you will be expected to use the *LEASEBUY* spreadsheet on the Applied Mathematics 30 CD. You may wish to modify this template to accommodate the information you collect. Then you will make a recommendation of whether it would be better for you to lease or buy the vehicle of your choice.

Turn to pages 182 and 183 of the textbook and read "Lease or Buy a Car?" Complete exercises 1 to 5 of "Lease or Buy a Car?" and keep a copy of your responses in the project section of your mathematics binder. You will need them to help you complete the questions in the Module Project section of Assignment Booklet 4B.

Before you continue your module project, critique the sample project titled "Financing Your Car" in exercise 11 of Part C of "What Should I Be Able to Do?" on page 205 of the textbook.

Compare your response with the suggested answer in the Appendix, Module Project, page 126.

Module Project

Now that you have more insight into the module project, revise your answers. Then complete the Module 4 project, Lease or Buy a Car?

You may use your responses from the textbook exercises on page 150 and pages 182 and 183 to help you complete the project. (Your responses should be in the project section of your mathematics binder.)

Turn to pages 21 to 25 of Assignment Booklet 4B and complete the module project.

MODULE SUMMARY

In this module, you explored compound interest, investing regularly, investment portfolios, how to shelter income from taxes by contributing to an RRSP, vehicle leases, and house mortgages. You used the TVM Solver on your graphing calculator and a spreadsheet program to help you sort through the facts and make judgments about particular financial decisions.

Some of the skills you acquired in this module should assist you in the future, like whether it is better for you to lease or purchase a vehicle or whether it is better for you to rent or buy a home. Becoming financially independent and relying on your own resource can only be gained through experience, knowledge of how investments work, and deciding whether the advice you receive from others is sound and appropriate for you!

APPENDIX

Glossary
Calculator Functions
Suggested Answers
Image Credits

Glossary

amortize: make payments to pay off a mortgage

amortization period: the length of time required to pay off a mortgage

capital gain: a gain in the value of property or equities

capital loss: a loss in the value of property or equities

compound interest: interest added to principal at the end of each interest period

effective rate of interest: the annual rate of interest that produces the same interest as a nominal rate compounded more frequently than once a year

equity: a share in the assets of a company

equities: stocks or mutual funds

equivalent rates of interest: two interest rates with different compounding periods that yield the same interest

mortgage: a loan on property where the property is held as collateral

mortgage rate: interest rate on a mortgage

nominal rate of interest: the stated rate of interest of a loan or deposit

portfolio: investment holdings

residual value: the option-to-purchase price of a vehicle at the end of its lease

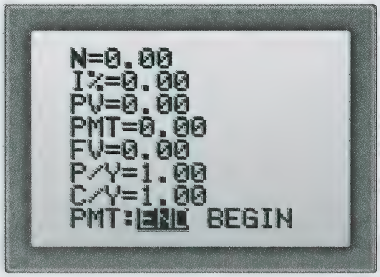
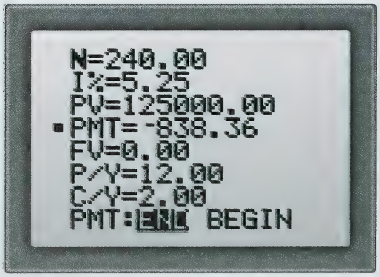
shares: certificates indicating partial ownership of a company

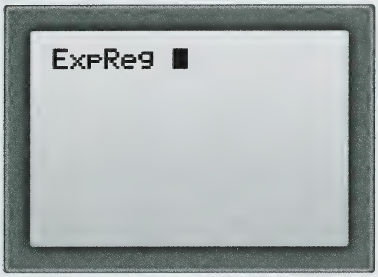
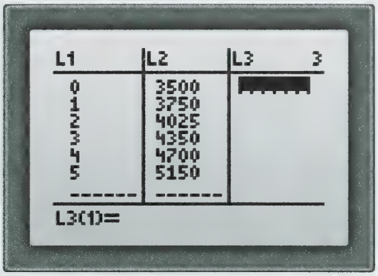
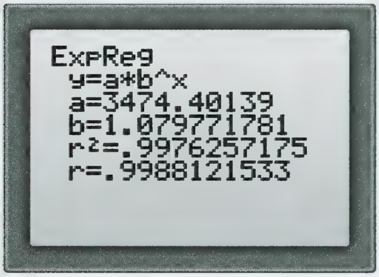
simple interest: interest on money borrowed or money lent calculated by multiplying the principal by the annual interest rate and by the time (in years)

stock: shares in a publicly traded company

template: a model that can be modified as required to solve problems, such as a prepared spreadsheet

Calculator Functions

Function	Keystrokes	Example
TVM Solver	2nd [FINANCE] 1 (1:TVMSolver…)  N=0.00 I%=0.00 PV=0.00 PMT=0.00 FV=0.00 P/Y=1.00 C/Y=1.00 PMT:END BEGIN	**Mortgage Payment**  N=240.00 I%=5.25 PV=125000.00 ▪PMT=-838.36 FV=0.00 P/Y=12.00 C/Y=2.00 PMT:END BEGIN

ExpReg	Select the CALC menu. STAT ▶ 0 (0:ExpReg)  ExpReg ■	 L1 L2 L3 3 0 3500 1 3750 2 4025 3 4350 4 4700 5 5150 L3(1)=  ExpReg y=a*b^x a=3474.40139 b=1.079771781 r²=.9976257175 r=.9988121533

To review how to obtain other calculator functions, refer to the Calculator folder on the Applied Mathematics 30 CD.

Suggested Answers

Activity 1: Investing Money

1. **Textbook exercises 1, 2, and 3 of "Practise Your Prior Skills," pp. 152 and 153**

1. a. $p=\$100$, $i=\dfrac{0.04}{2}=0.02$, and $n=3\times 2=6$

$$\begin{aligned}\therefore A&=P(1+i)^n\\&=100(1+0.02)^6\\&=\$112.62\end{aligned}$$

b. $p=\$100$, $i=\dfrac{0.06}{12}=0.005$, and $n=2\times 12=24$

$$\begin{aligned}\therefore A&=P(1+i)^n\\&=100(1+0.005)^{24}\\&=\$112.72\end{aligned}$$

2. a. $p=\$1000$, $i=0.06$, and $n=10$

$$\begin{aligned}\therefore A&=P(1+i)^n\\&=1000(1+0.06)^{10}\\&=\$1790.85\end{aligned}$$

b. $p=\$1000$, $i=\dfrac{0.085}{2}=0.0425$, and $n=10\times 2=20$

$$\begin{aligned}\therefore A&=P(1+i)^n\\&=1000(1+0.0425)^{20}\\&=\$2298.91\end{aligned}$$

c. $p = \$1000$, $i = \frac{0.0775}{12}$, and $n = 10 \times 12$

$= 120$

$$\begin{aligned} \therefore A &= P(1+i)^n \\ &= 1000\left(1 + \frac{0.0775}{12}\right)^{120} \\ &= \$2165.19 \end{aligned}$$

3. a. i. $$\begin{aligned} t &= \frac{72}{4} \\ &= 18 \end{aligned}$$

The doubling time at 4% is approximately 18 years.

ii. $$\begin{aligned} t &= \frac{72}{8} \\ &= 9 \end{aligned}$$

The doubling time at 8% is approximately 9 years.

iii. $$\begin{aligned} t &= \frac{72}{24} \\ &= 3 \end{aligned}$$

The doubling time at 24% is approximately 3 years.

b. i. $p = \$100$, $i = 0.04$, and $n = 18$

$$\begin{aligned} \therefore A &= P(1+i)^n \\ &= 100(1+0.04)^{18} \\ &= \$202.58 \end{aligned}$$

ii. $p = \$100$, $i = 0.08$, and $n = 9$

$$\begin{aligned} \therefore A &= P(1+i)^n \\ &= 100(1+0.08)^{9} \\ &= \$199.90 \end{aligned}$$

iii. $p = \$100$, $i = 0.24$, and $n = 3$

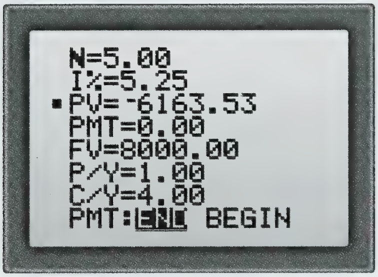
$$\begin{aligned} \therefore A &= P(1+i)^n \\ &= 100(1+0.24)^{3} \\ &= \$190.66 \end{aligned}$$

c. The rule of 72 gives a good estimate for the doubling times, particularly for interest rates 8% or lower.

2. Textbook exercise 4 of "Practise Your Prior Skills," p. 153

4. Use the TVM Solver.

$N = 5, I\% = 5.25, PV = ?, PMT = 0, FV = 8000, P/Y = 1.00$, and $C/Y = 4.00$

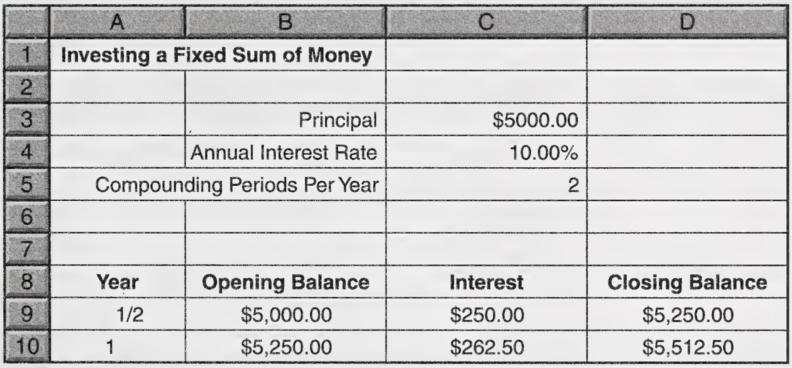


```
 N=5.00
 I%=5.25
▪PV=-6163.53
 PMT=0.00
 FV=8000.00
 P/Y=1.00
 C/Y=4.00
 PMT:END BEGIN
```

To have $8000 in 5 years time, $6163.53 must be invested today at rate of 5.25%, compounded quarterly.

3. Textbook exercises 1 to 5 of "Investigation 1: Watching Money Grow," pp. 153 and 154

1. a. Display cell C3 as currency and cell C4 as a percent.
b. Display cells A9 and A10 as a fraction.
c. Display cells B9 and B10 as currency.
d. Display cells C9 and C10 as currency.
e. Display cells D9 and D10 as currency.



	A	B	C	D
1	**Investing a Fixed Sum of Money**			
2				
3		Principal	$5000.00	
4		Annual Interest Rate	10.00%	
5		Compounding Periods Per Year	2	
6				
7				
8	**Year**	**Opening Balance**	**Interest**	**Closing Balance**
9	1/2	$5,000.00	$250.00	$5,250.00
10	1	$5,250.00	$262.50	$5,512.50

2. Cell A9 displays the first interest period; it occurs at the end of 6 months or $\frac{1}{2}$ year.

$$\frac{1}{2} = 1/\$C\$5, \text{ where } \$C\$5 = 2$$

Therefore, A9=1/$C$5.

Cell B9 is the original principal. Therefore, B9=C3.

Cell C9 is the interest earned on the opening balance of the first interest period. Because interest is calculated more than once a year, you must multiply B9 by the annual rate divided by the number of compounding periods per year. Therefore, C9=B9*$C$4/$C$5.

Cell D9 is the closing balance, which is the opening balance plus interest. Therefore, D9=B9+C9.

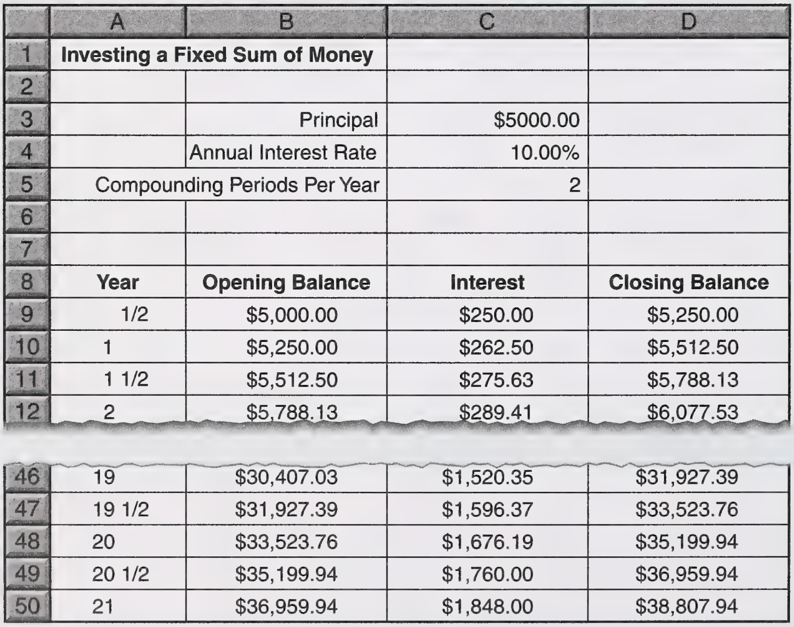
Cell A10 is the end of the second interest period, which is A9 plus one divided by the number of compounding periods. Therefore, A10=A9+1/$C$5.

Cell B10 is the opening balance for the second interest period. This amount is the same as the closing balance for the first interest period. Therefore, B10=D9.

Cell C10 is the interest calculated on the opening balance of the second interest period. Therefore, C10=B10*$C$4/$C$5.

Cell D10 is the closing balance for the second interest period. The amount in this cell is opening balance (B10) plus interest (C10). Therefore, D10=B10+C10.

3.



	A	B	C	D
1	**Investing a Fixed Sum of Money**			
2				
3		Principal	$5000.00	
4		Annual Interest Rate	10.00%	
5	Compounding Periods Per Year		2	
6				
7				
8	**Year**	**Opening Balance**	**Interest**	**Closing Balance**
9	1/2	$5,000.00	$250.00	$5,250.00
10	1	$5,250.00	$262.50	$5,512.50
11	1 1/2	$5,512.50	$275.63	$5,788.13
12	2	$5,788.13	$289.41	$6,077.53
46	19	$30,407.03	$1,520.35	$31,927.39
47	19 1/2	$31,927.39	$1,596.37	$33,523.76
48	20	$33,523.76	$1,676.19	$35,199.94
49	20 1/2	$35,199.94	$1,760.00	$36,959.94
50	21	$36,959.94	$1,848.00	$38,807.94

4. The interest rate used in each interest period is $\frac{10\%}{2} = 5\%$.

Closing Balance of First Interest Period

$$\begin{aligned} D9 &= \$5000.00 + 5\% \text{ of } \$5000.00 \\ &= \$5000.00 + 0.05(\$5000.00) \\ &= \$5000.00 + \$250.00 \\ &= \$5250.00 \end{aligned}$$

Closing Balance of Second Interest Period

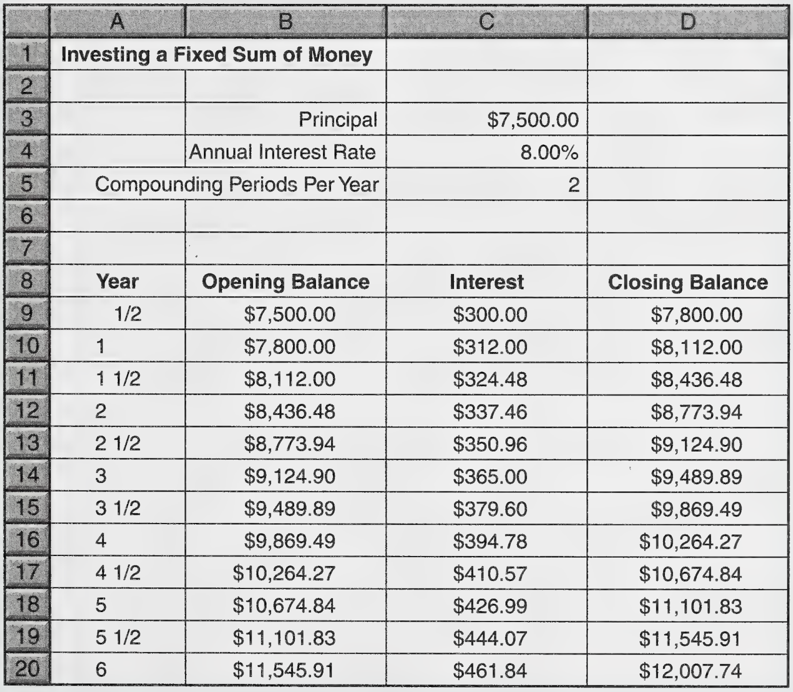
$$\begin{aligned} D10 &= \$5250.00 + 5\% \text{ of } \$5250.00 \\ &= \$5250.00 + 0.05(\$5250.00) \\ &= \$5250.00 + \$262.50 \\ &= \$5512.50 \end{aligned}$$

Closing Balance of Third Interest Period

$$\begin{aligned} D11 &= \$5512.50 + 5\% \text{ of } \$5512.50 \\ &= \$5512.50 + 0.05(\$5512.50) \\ &= \$5512.50 + \$275.63 \\ &= \$5788.13 \end{aligned}$$

These values are the same as those in the spreadsheet.

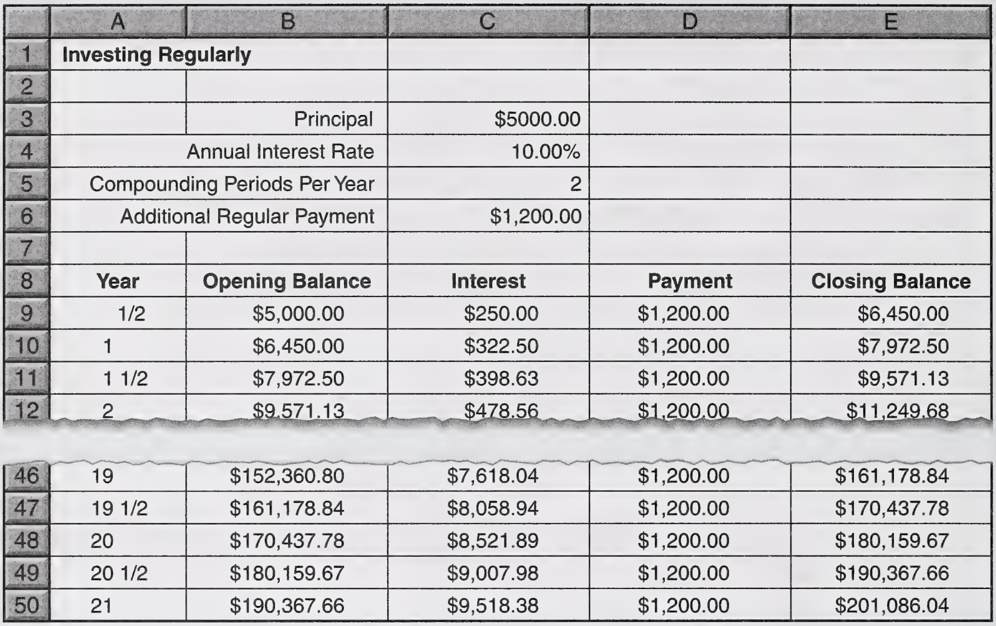
5. Enter 7500 in cell C3, 8% in cell C4, and 2 in cell C5 (if needed).



	A	B	C	D
1	**Investing a Fixed Sum of Money**			
2				
3		Principal	$7,500.00	
4		Annual Interest Rate	8.00%	
5	Compounding Periods Per Year		2	
6				
7				
8	**Year**	**Opening Balance**	**Interest**	**Closing Balance**
9	1/2	$7,500.00	$300.00	$7,800.00
10	1	$7,800.00	$312.00	$8,112.00
11	1 1/2	$8,112.00	$324.48	$8,436.48
12	2	$8,436.48	$337.46	$8,773.94
13	2 1/2	$8,773.94	$350.96	$9,124.90
14	3	$9,124.90	$365.00	$9,489.89
15	3 1/2	$9,489.89	$379.60	$9,869.49
16	4	$9,869.49	$394.78	$10,264.27
17	4 1/2	$10,264.27	$410.57	$10,674.84
18	5	$10,674.84	$426.99	$11,101.83
19	5 1/2	$11,101.83	$444.07	$11,545.91
20	6	$11,545.91	$461.84	$12,007.74

4. Textbook exercises 1 to 6 of "Investigation 2: Investing Regularly," pp. 155 and 156

1. to 3. Your spreadsheet should look like the following.



	A	B	C	D	E
1	**Investing Regularly**				
2					
3		Principal	$5000.00		
4		Annual Interest Rate	10.00%		
5		Compounding Periods Per Year	2		
6		Additional Regular Payment	$1,200.00		
7					
8	**Year**	**Opening Balance**	**Interest**	**Payment**	**Closing Balance**
9	1/2	$5,000.00	$250.00	$1,200.00	$6,450.00
10	1	$6,450.00	$322.50	$1,200.00	$7,972.50
11	1 1/2	$7,972.50	$398.63	$1,200.00	$9,571.13
12	2	$9,571.13	$478.56	$1,200.00	$11,249.68
46	19	$152,360.80	$7,618.04	$1,200.00	$161,178.84
47	19 1/2	$161,178.84	$8,058.94	$1,200.00	$170,437.78
48	20	$170,437.78	$8,521.89	$1,200.00	$180,159.67
49	20 1/2	$180,159.67	$9,007.98	$1,200.00	$190,367.66
50	21	$190,367.66	$9,518.38	$1,200.00	$201,086.04

4. The interest rate used in every interest period is $\frac{10\%}{2} = 5\%$.

Closing Balance of First Interest Period

$$\begin{aligned}\text{E}9 &= \$5000.00 + 0.05(\$5000.00) + \$1200.00\\ &= \$5000.00 + \$250.00 + \$1200.00\\ &= \$6450.00\end{aligned}$$

Closing Balance of Second Interest Period

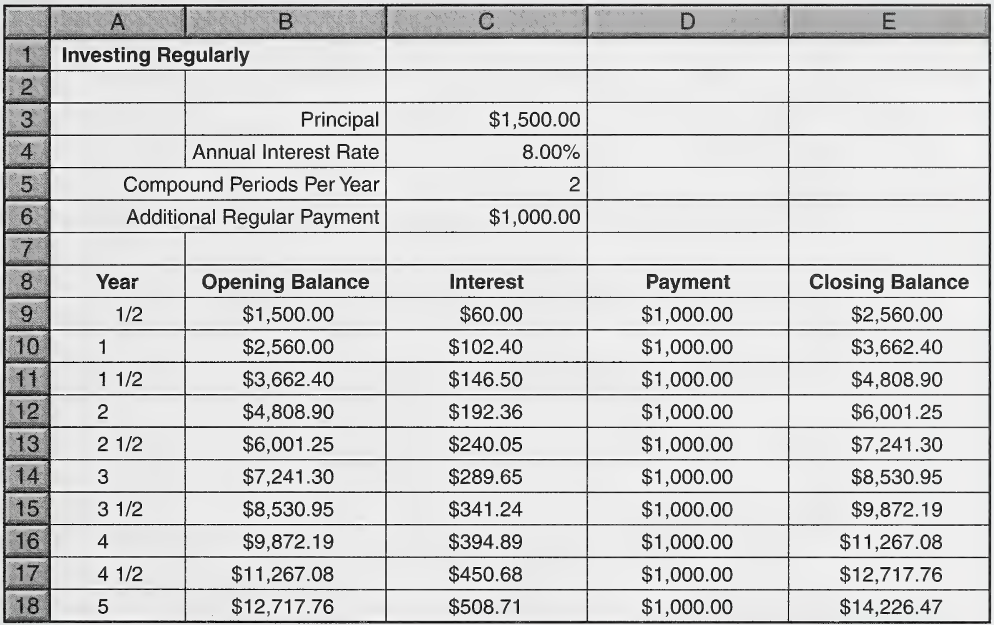
$$\begin{aligned}\text{E}10 &= \$6450.00 + 0.05(\$6450.00) + \$1200.00\\ &= \$6450.00 + \$322.50 + \$1200.00\\ &= \$7972.50\end{aligned}$$

Closing Balance of Third Interest Period

$$\begin{aligned} E11 &= \$7972.50 + 0.05(\$7972.50) + \$1200.00 \\ &= \$7972.50 + \$398.63 + \$1200.00 \\ &= \$9571.13 \end{aligned}$$

These values are the same as those in the spreadsheet.

5. Enter $1500 in cell C3, 8% in cell C4, 2 in cell C5 (if needed), and $1000 in cell C6.

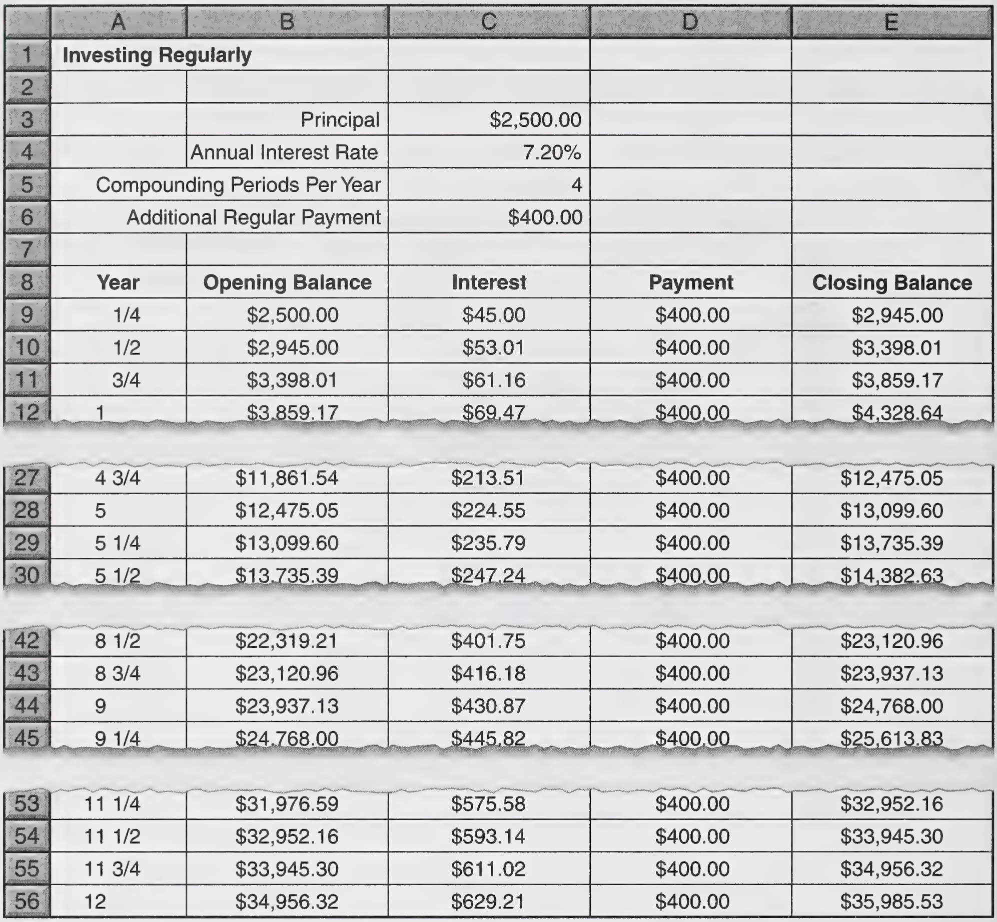


	A	B	C	D	E
1	**Investing Regularly**				
2					
3		Principal	$1,500.00		
4		Annual Interest Rate	8.00%		
5	Compound Periods Per Year		2		
6	Additional Regular Payment		$1,000.00		
7					
8	**Year**	**Opening Balance**	**Interest**	**Payment**	**Closing Balance**
9	1/2	$1,500.00	$60.00	$1,000.00	$2,560.00
10	1	$2,560.00	$102.40	$1,000.00	$3,662.40
11	1 1/2	$3,662.40	$146.50	$1,000.00	$4,808.90
12	2	$4,808.90	$192.36	$1,000.00	$6,001.25
13	2 1/2	$6,001.25	$240.05	$1,000.00	$7,241.30
14	3	$7,241.30	$289.65	$1,000.00	$8,530.95
15	3 1/2	$8,530.95	$341.24	$1,000.00	$9,872.19
16	4	$9,872.19	$394.89	$1,000.00	$11,267.08
17	4 1/2	$11,267.08	$450.68	$1,000.00	$12,717.76
18	5	$12,717.76	$508.71	$1,000.00	$14,226.47

The final balance after 5 years is $14 226.47.

Activity 1 (continued)

6. Enter $2500 in cell C3, 7.2% in cell C4, 4 in cell C5, and $400 in cell C6.



	A	B	C	D	E
1	**Investing Regularly**				
2					
3		Principal	$2,500.00		
4		Annual Interest Rate	7.20%		
5		Compounding Periods Per Year	4		
6		Additional Regular Payment	$400.00		
7					
8	**Year**	**Opening Balance**	**Interest**	**Payment**	**Closing Balance**
9	1/4	$2,500.00	$45.00	$400.00	$2,945.00
10	1/2	$2,945.00	$53.01	$400.00	$3,398.01
11	3/4	$3,398.01	$61.16	$400.00	$3,859.17
12	1	$3,859.17	$69.47	$400.00	$4,328.64
27	4 3/4	$11,861.54	$213.51	$400.00	$12,475.05
28	5	$12,475.05	$224.55	$400.00	$13,099.60
29	5 1/4	$13,099.60	$235.79	$400.00	$13,735.39
30	5 1/2	$13,735.39	$247.24	$400.00	$14,382.63
42	8 1/2	$22,319.21	$401.75	$400.00	$23,120.96
43	8 3/4	$23,120.96	$416.18	$400.00	$23,937.13
44	9	$23,937.13	$430.87	$400.00	$24,768.00
45	9 1/4	$24,768.00	$445.82	$400.00	$25,613.83
53	11 1/4	$31,976.59	$575.58	$400.00	$32,952.16
54	11 1/2	$32,952.16	$593.14	$400.00	$33,945.30
55	11 3/4	$33,945.30	$611.02	$400.00	$34,956.32
56	12	$34,956.32	$629.21	$400.00	$35,985.53

It will take 12 years to save at least $35 000.

5. a. **Textbook exercises 1, 2, and 3 of "Discussing the Ideas," p. 157**

1. When you create a spreadsheet and name cells, you can use these names to set up cell formulas elsewhere in your spreadsheet. For example, in Investigation 1, you could have named cell C4 *Rate* and used this variable in the formula for cell C9, making it C9=B9*Rate/$C$5. You must be careful, though, if there are several different rates in the spreadsheet. This may cause confusion for someone using your spreadsheet if you do not use different names.

Not all spreadsheets allow naming cells with meaningful names like *Principal* or *Rate-of-Return*. In these spreadsheets you have to use absolute addressing to allow formulas to be copied into additional cells.

2. A spreadsheet is useful if you want to save your analysis, use the spreadsheet as a template to solve similar problems, display your results in table form, investigate intermediate values, or print your results. However, spreadsheets can be complicated to program.

   A calculator is often preferred when you need to solve a problem for a single answer or create a short table that you do not need to save for future use.

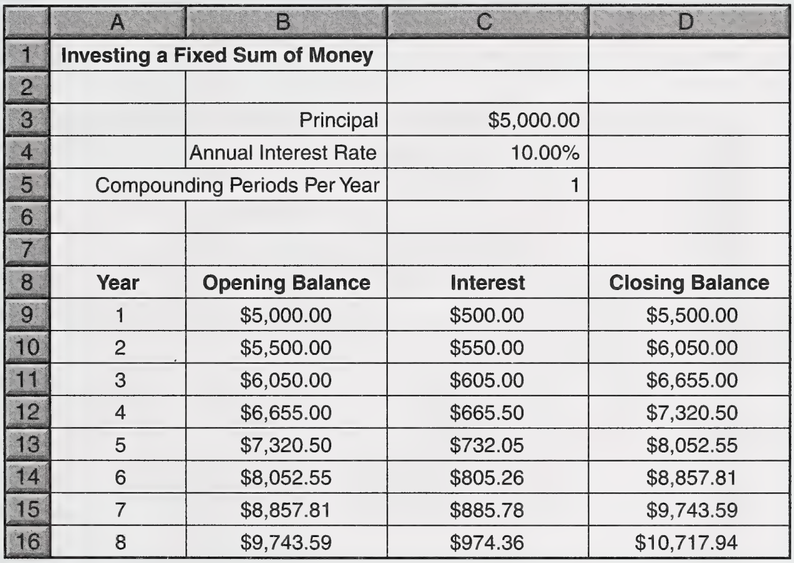
3. In the example, the value of PMT is negative because this is money you are depositing and not receiving. The value of PMT would be positive if you received this payment rather than paid it.

b. Textbook exercises 1, 2, 3, 5, 6, and 7 of "Exercises: Checking Your Skills," pp. 158 and 159

1. a. This screen displays the amount you would have after 2 years if you deposited $275 every month in an account bearing 8% interest, compounded semi-annually.

   b. This screen displays the amount you could withdraw every month for 2 years if $5000 was initially deposited into an account bearing 8% interest, compounded monthly.

2. a. **Method 1: Using a Spreadsheet**

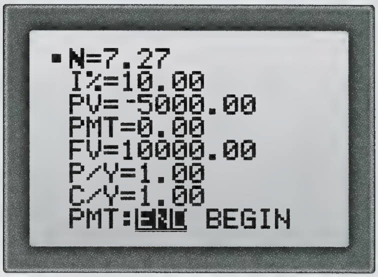


	A	B	C	D
1	**Investing a Fixed Sum of Money**			
2				
3		Principal	$5,000.00	
4		Annual Interest Rate	10.00%	
5	Compounding Periods Per Year		1	
6				
7				
8	**Year**	**Opening Balance**	**Interest**	**Closing Balance**
9	1	$5,000.00	$500.00	$5,500.00
10	2	$5,500.00	$550.00	$6,050.00
11	3	$6,050.00	$605.00	$6,655.00
12	4	$6,655.00	$665.50	$7,320.50
13	5	$7,320.50	$732.05	$8,052.55
14	6	$8,052.55	$805.26	$8,857.81
15	7	$8,857.81	$885.78	$9,743.59
16	8	$9,743.59	$974.36	$10,717.94

It will take 8 years for the investment to at least double.

Method 2: Using the TVM Solver

Press 2nd [FINANCE] 1 to select the TVM Solver. Enter the known values, and determine the number of years, N.



```
▪N=7.27
 I%=10.00
 PV=-5000.00
 PMT=0.00
 FV=10000.00
 P/Y=1.00
 C/Y=1.00
 PMT:END BEGIN
```

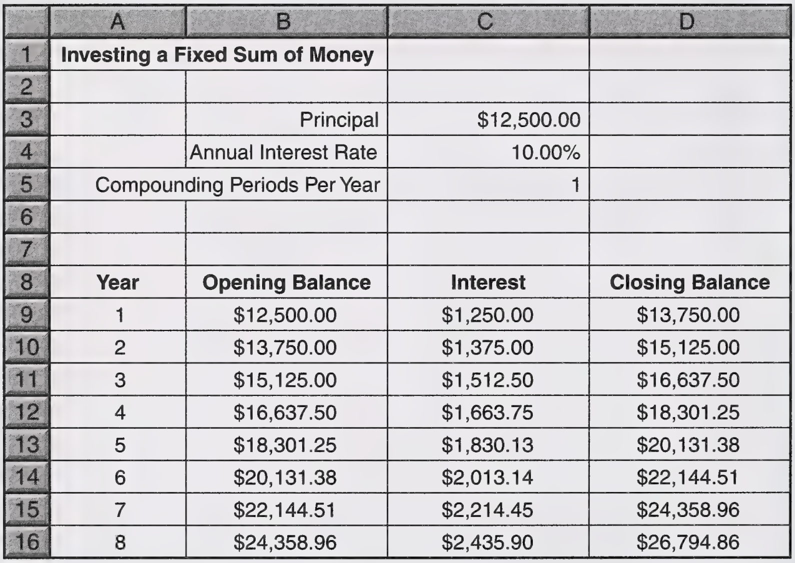
Because N is greater than 7 and because interest is calculated annually, it will take 8 years for the investment to at least double.

According to Rule of 72,

$$
\begin{aligned}
t &= \frac{72}{10} \\
&= 7.2
\end{aligned}
$$

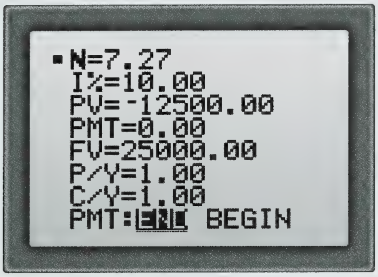
Therefore, the answer does follow the Rule of 72.

b. Method 1: Using a Spreadsheet



	A	B	C	D
1	**Investing a Fixed Sum of Money**			
2				
3		Principal	$12,500.00	
4		Annual Interest Rate	10.00%	
5	Compounding Periods Per Year		1	
6				
7				
8	**Year**	**Opening Balance**	**Interest**	**Closing Balance**
9	1	$12,500.00	$1,250.00	$13,750.00
10	2	$13,750.00	$1,375.00	$15,125.00
11	3	$15,125.00	$1,512.50	$16,637.50
12	4	$16,637.50	$1,663.75	$18,301.25
13	5	$18,301.25	$1,830.13	$20,131.38
14	6	$20,131.38	$2,013.14	$22,144.51
15	7	$22,144.51	$2,214.45	$24,358.96
16	8	$24,358.96	$2,435.90	$26,794.86

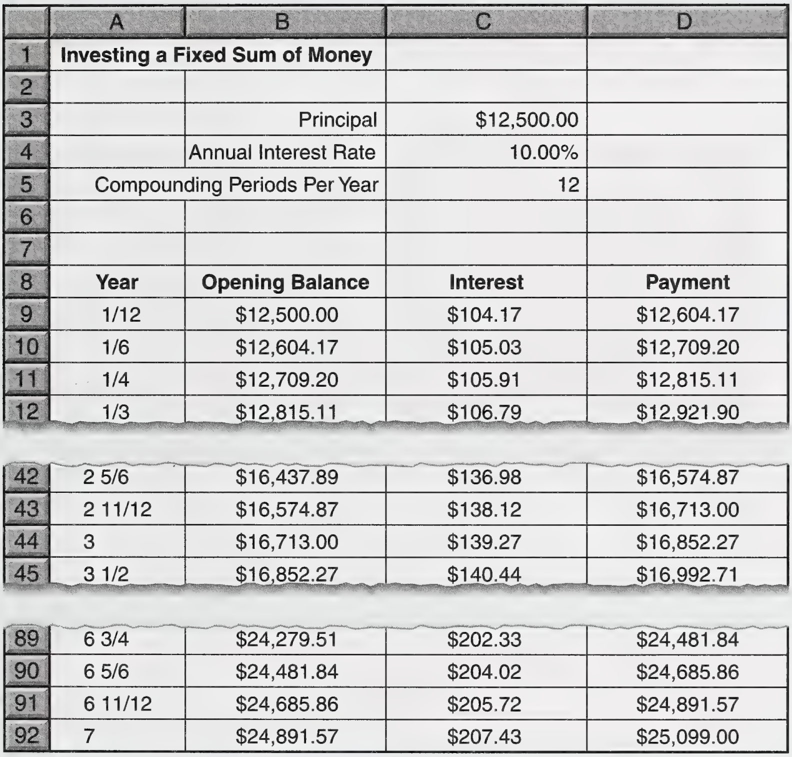
Method 2: Using the TVM Solver



```
▪N=7.27
 I%=10.00
 PV=-12500.00
 PMT=0.00
 FV=25000.00
 P/Y=1.00
 C/Y=1.00
 PMT:END BEGIN
```

The amount of the principal does not affect the doubling period.

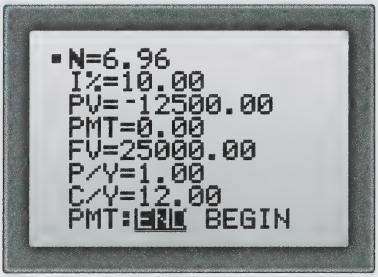
c. **Method 1: Using a Spreadsheet**



	A	B	C	D
1	**Investing a Fixed Sum of Money**			
2				
3		Principal	$12,500.00	
4		Annual Interest Rate	10.00%	
5	Compounding Periods Per Year		12	
6				
7				
8	**Year**	**Opening Balance**	**Interest**	**Payment**
9	1/12	$12,500.00	$104.17	$12,604.17
10	1/6	$12,604.17	$105.03	$12,709.20
11	1/4	$12,709.20	$105.91	$12,815.11
12	1/3	$12,815.11	$106.79	$12,921.90
42	2 5/6	$16,437.89	$136.98	$16,574.87
43	2 11/12	$16,574.87	$138.12	$16,713.00
44	3	$16,713.00	$139.27	$16,852.27
45	3 1/2	$16,852.27	$140.44	$16,992.71
89	6 3/4	$24,279.51	$202.33	$24,481.84
90	6 5/6	$24,481.84	$204.02	$24,685.86
91	6 11/12	$24,685.86	$205.72	$24,891.57
92	7	$24,891.57	$207.43	$25,099.00

Activity 1 (continued)

Method 2: Using the TVM Solver

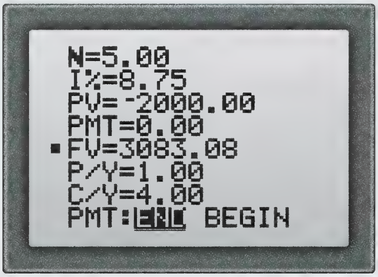


```
▪N=6.96
 I%=10.00
 PV=-12500.00
 PMT=0.00
 FV=25000.00
 P/Y=1.00
 C/Y=12.00
 PMT:END BEGIN
```

The doubling time reduces from 8 years to 7 years when the interest rate is compounded monthly instead of annually.

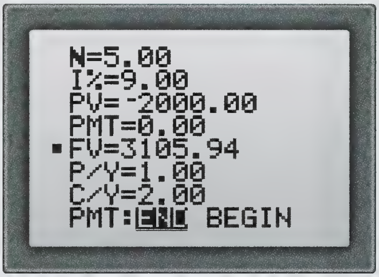
3. Use the TVM Solver to determine the future value of the investment after 5 years for each option.

GIC



```
 N=5.00
 I%=8.75
 PV=-2000.00
 PMT=0.00
▪FV=3083.08
 P/Y=1.00
 C/Y=4.00
 PMT:END BEGIN
```

Savings Bonds



```
 N=5.00
 I%=9.00
 PV=-2000.00
 PMT=0.00
▪FV=3105.94
 P/Y=1.00
 C/Y=2.00
 PMT:END BEGIN
```

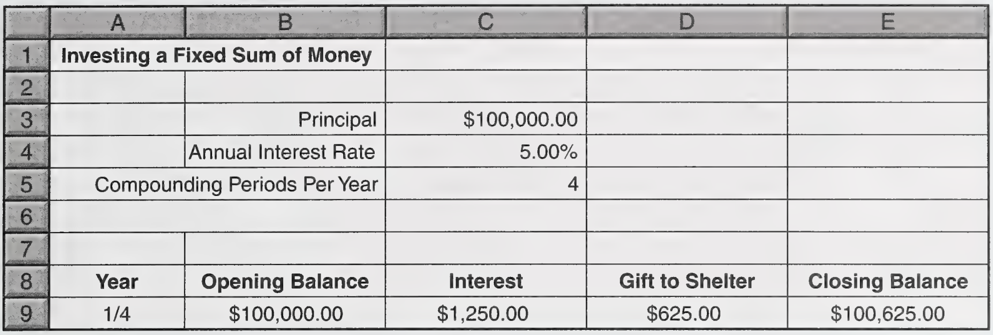
The GIC will yield a future value of $3083.08 after 5 years, and the savings bonds will yield a future value of $3105.94. Therefore, the savings bonds will earn $22.86 more in the five-year term.

5. a. Using the spreadsheet *Investing a Fixed Sum of Money* as a template, insert a column between “Interest” and “Closing Balance.” Name the new column “Gift to Shelter.”

Now, enter the following into the cells listed:

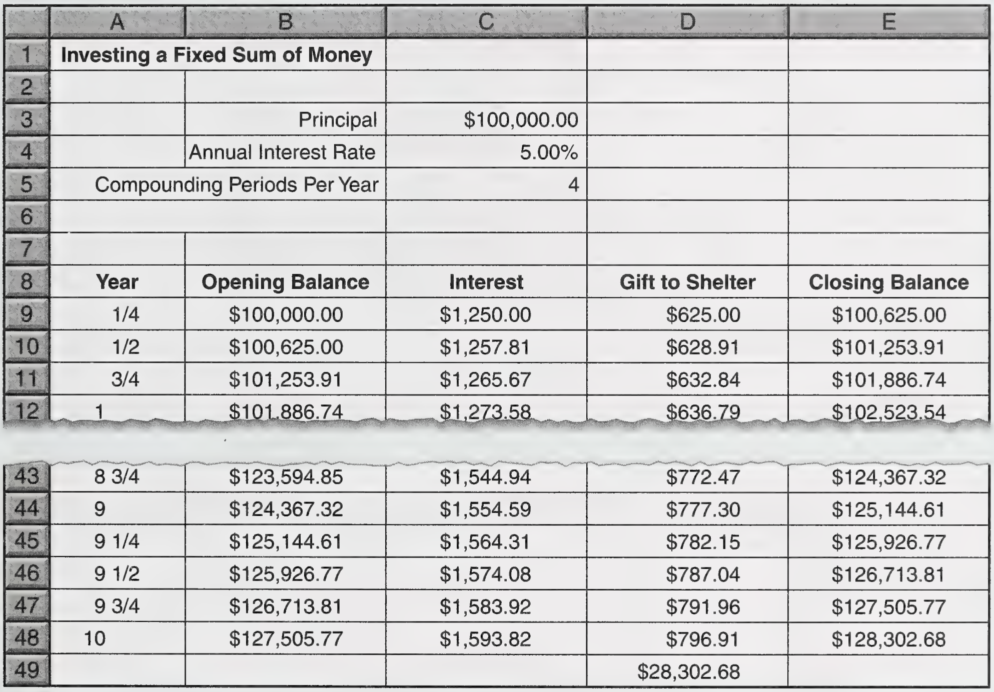
- C3: 100 000
- C4: 5%
- C5: 4
- D9: C9/2
- D10: =B9+C9–D9

Your spreadsheet should look like the following.



	A	B	C	D	E
1	**Investing a Fixed Sum of Money**				
2					
3		Principal	$100,000.00		
4		Annual Interest Rate	5.00%		
5		Compounding Periods Per Year	4		
6					
7					
8	**Year**	**Opening Balance**	**Interest**	**Gift to Shelter**	**Closing Balance**
9	1/4	$100,000.00	$1,250.00	$625.00	$100,625.00

b. Select cells A9 to E9, and fill down to 10 years. Then find the sum of column D to determine the total amount that was given to the animal shelter after 10 years.



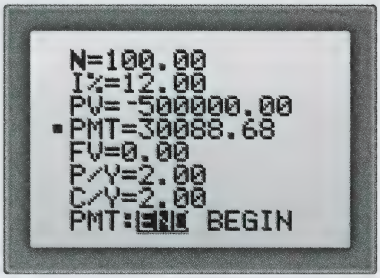
	A	B	C	D	E
1	**Investing a Fixed Sum of Money**				
2					
3		Principal	$100,000.00		
4		Annual Interest Rate	5.00%		
5		Compounding Periods Per Year	4		
6					
7					
8	**Year**	**Opening Balance**	**Interest**	**Gift to Shelter**	**Closing Balance**
9	1/4	$100,000.00	$1,250.00	$625.00	$100,625.00
10	1/2	$100,625.00	$1,257.81	$628.91	$101,253.91
11	3/4	$101,253.91	$1,265.67	$632.84	$101,886.74
12	1	$101,886.74	$1,273.58	$636.79	$102,523.54
43	8 3/4	$123,594.85	$1,544.94	$772.47	$124,367.32
44	9	$124,367.32	$1,554.59	$777.30	$125,144.61
45	9 1/4	$125,144.61	$1,564.31	$782.15	$125,926.77
46	9 1/2	$125,926.77	$1,574.08	$787.04	$126,713.81
47	9 3/4	$126,713.81	$1,583.92	$791.96	$127,505.77
48	10	$127,505.77	$1,593.82	$796.91	$128,302.68
49				$28,302.68	

After a 10-year period, the shelter will have received $28 302.68.

6. Use the TVM Solver.

> You will be receiving 2 payments per year for 50 years, so $N = 2 \times 50 = 100$.
> The annual interest rate is 12%, so $I\% = 12$.
> The initial deposit is \$500 000, so $PV = -500\ 000$.
> The future value will be \$0 after 50 years, so $FV = 0$.
> You will receive 2 payments per year, so $P/Y = 2$.
> There are 2 compounding periods per year, so $C/Y = 2$.
> The payments are made at the end of each period.

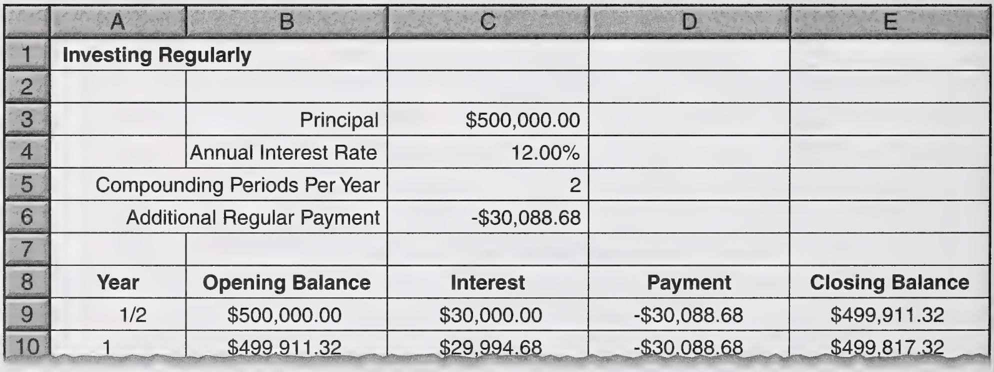
Position the cursor on PMT, and press ALPHA [SOLVE].



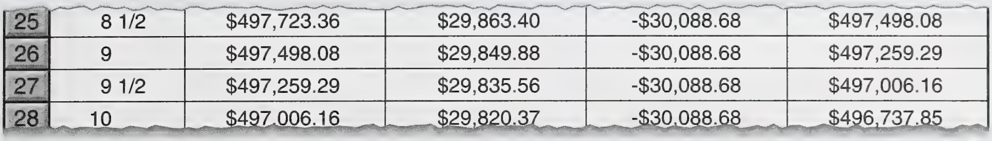
```
 N=100.00
 I%=12.00
 PV=-500000.00
▪PMT=30088.68
 FV=0.00
 P/Y=2.00
 C/Y=2.00
 PMT:END BEGIN
```

You will receive \$30 088.68 per period.

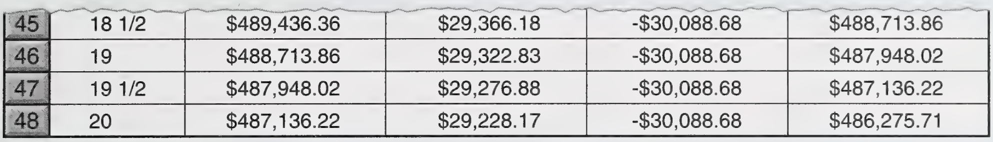
7. a. Use the spreadsheet *Investing Regularly*. Simply enter the regular payment in cell C6 as a negative value because it is money you are withdrawing.



	A	B	C	D	E
1	**Investing Regularly**				
2					
3		Principal	\$500,000.00		
4		Annual Interest Rate	12.00%		
5		Compounding Periods Per Year	2		
6		Additional Regular Payment	-\$30,088.68		
7					
8	**Year**	**Opening Balance**	**Interest**	**Payment**	**Closing Balance**
9	1/2	\$500,000.00	\$30,000.00	-\$30,088.68	\$499,911.32
10	1	\$499,911.32	\$29,994.68	-\$30,088.68	\$499,817.32



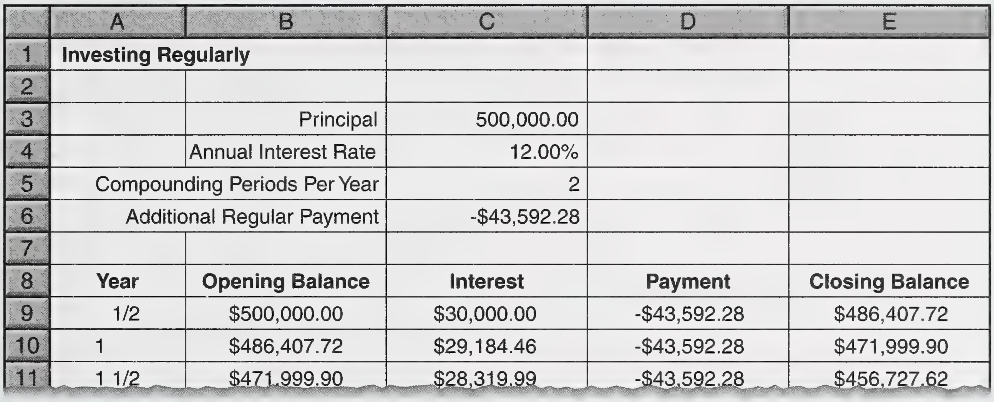
25	8 1/2	$497,723.36	$29,863.40	-$30,088.68	$497,498.08
26	9	$497,498.08	$29,849.88	-$30,088.68	$497,259.29
27	9 1/2	$497,259.29	$29,835.56	-$30,088.68	$497,006.16
28	10	$497,006.16	$29,820.37	-$30,088.68	$496,737.85



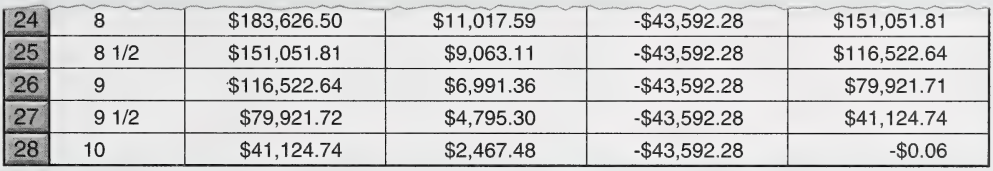
45	18 1/2	$489,436.36	$29,366.18	-$30,088.68	$488,713.86
46	19	$488,713.86	$29,322.83	-$30,088.68	$487,948.02
47	19 1/2	$487,948.02	$29,276.88	-$30,088.68	$487,136.22
48	20	$487,136.22	$29,228.17	-$30,088.68	$486,275.71

There will be $496 737.85 left after 10 years and $486 275.71 left after 20 years.

b. Use guess and check.



	A	B	C	D	E
1	**Investing Regularly**				
2					
3		Principal	500,000.00		
4		Annual Interest Rate	12.00%		
5		Compounding Periods Per Year	2		
6		Additional Regular Payment	-$43,592.28		
7					
8	**Year**	**Opening Balance**	**Interest**	**Payment**	**Closing Balance**
9	1/2	$500,000.00	$30,000.00	-$43,592.28	$486,407.72
10	1	$486,407.72	$29,184.46	-$43,592.28	$471,999.90
11	1 1/2	$471,999.90	$28,319.99	-$43,592.28	$456,727.62

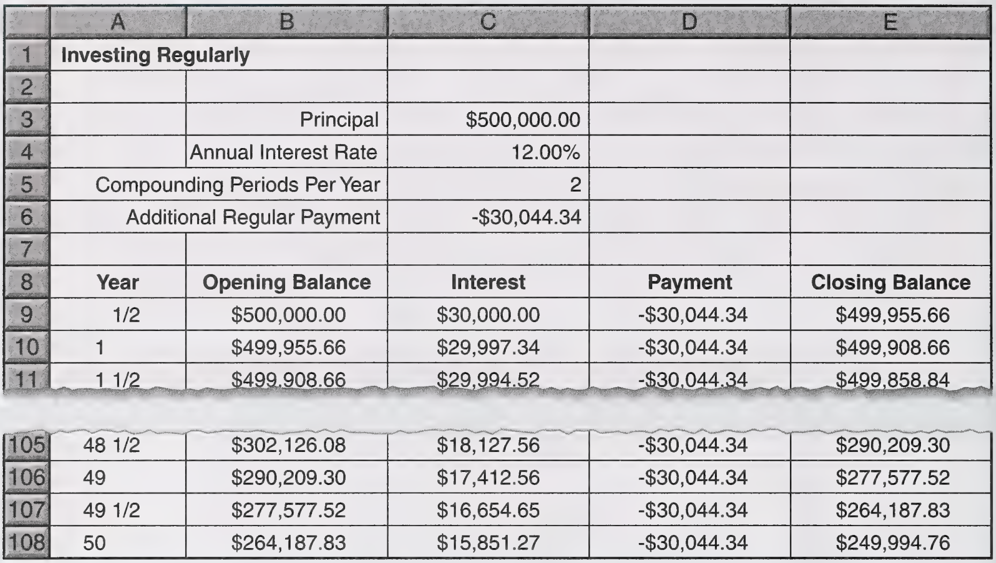


24	8	$183,626.50	$11,017.59	-$43,592.28	$151,051.81
25	8 1/2	$151,051.81	$9,063.11	-$43,592.28	$116,522.64
26	9	$116,522.64	$6,991.36	-$43,592.28	$79,921.71
27	9 1/2	$79,921.72	$4,795.30	-$43,592.28	$41,124.74
28	10	$41,124.74	$2,467.48	-$43,592.28	-$0.06

To spend the money within 10 years, you will have to withdraw $43 592.28 every six months.

Activity 1 (continued)

c.



	A	B	C	D	E
1	**Investing Regularly**				
2					
3		Principal	$500,000.00		
4		Annual Interest Rate	12.00%		
5		Compounding Periods Per Year	2		
6		Additional Regular Payment	-$30,044.34		
7					
8	**Year**	**Opening Balance**	**Interest**	**Payment**	**Closing Balance**
9	1/2	$500,000.00	$30,000.00	-$30,044.34	$499,955.66
10	1	$499,955.66	$29,997.34	-$30,044.34	$499,908.66
11	1 1/2	$499,908.66	$29,994.52	-$30,044.34	$499,858.84
105	48 1/2	$302,126.08	$18,127.56	-$30,044.34	$290,209.30
106	49	$290,209.30	$17,412.56	-$30,044.34	$277,577.52
107	49 1/2	$277,577.52	$16,654.65	-$30,044.34	$264,187.83
108	50	$264,187.83	$15,851.27	-$30,044.34	$249,994.76

To leave about $250 000 from your winnings after 50 years, you will be able to withdraw $30 044.34 every six months.

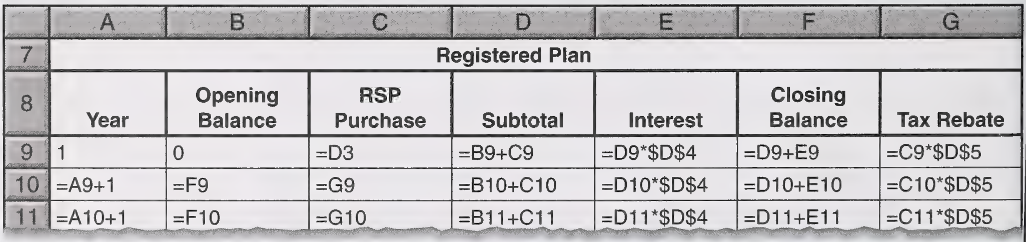
6. Textbook exercise "Communicating the Ideas," p. 159

It is better to invest small amounts monthly rather than an equivalent amount at the end of the year because you will receive interest on your monthly deposits. Over the years your money will grow faster because it is compounded more often. Examples could be illustrated using the TVM Solver on your graphing calculator or using a spreadsheet.

Activity 2: Investing for the Future

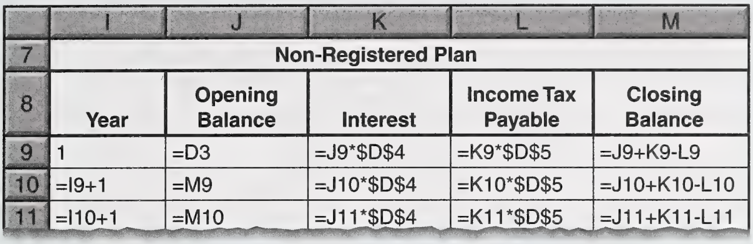
1. Textbook exercises 1 to 6 of "Investigation 1: The Power of RRSPs," pp. 160 and 161

1. to 3. The formulas for the Registered Plan section should be similar to the ones shown in the partial spreadsheet given.



	A	B	C	D	E	F	G
7	**Registered Plan**						
8	**Year**	**Opening Balance**	**RSP Purchase**	**Subtotal**	**Interest**	**Closing Balance**	**Tax Rebate**
9	1	0	=D3	=B9+C9	=D9*$D$4	=D9+E9	=C9*$D$5
10	=A9+1	=F9	=G9	=B10+C10	=D10*$D$4	=D10+E10	=C10*$D$5
11	=A10+1	=F10	=G10	=B11+C11	=D11*$D$4	=D11+E11	=C11*$D$5

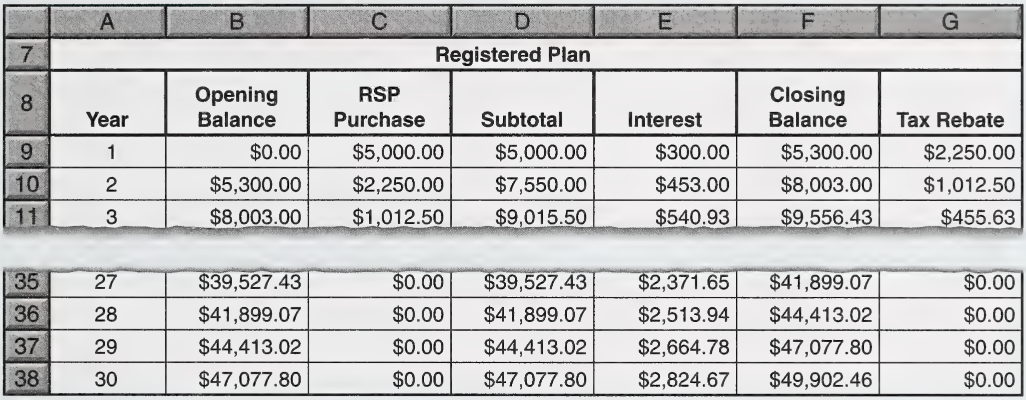
4. The formulas for the Non-Registered Plan should be similar to the ones shown in the partial spreadsheet given.



	I	J	K	L	M
7	Non-Registered Plan				
8	Year	Opening Balance	Interest	Income Tax Payable	Closing Balance
9	1	=D3	=J9*$D$4	=K9*$D$5	=J9+K9-L9
10	=I9+1	=M9	=J10*$D$4	=K10*$D$5	=J10+K10-L10
11	=I10+1	=M10	=J11*$D$4	=K11*$D$5	=J11+K11-L11

5. The formulas in cells A10, B10, C10, I10, and J10 are different than their corresponding formulas in row 9; the others are equivalent.

6. **Registered Plan**



	A	B	C	D	E	F	G
7	Registered Plan						
8	Year	Opening Balance	RSP Purchase	Subtotal	Interest	Closing Balance	Tax Rebate
9	1	$0.00	$5,000.00	$5,000.00	$300.00	$5,300.00	$2,250.00
10	2	$5,300.00	$2,250.00	$7,550.00	$453.00	$8,003.00	$1,012.50
11	3	$8,003.00	$1,012.50	$9,015.50	$540.93	$9,556.43	$455.63
35	27	$39,527.43	$0.00	$39,527.43	$2,371.65	$41,899.07	$0.00
36	28	$41,899.07	$0.00	$41,899.07	$2,513.94	$44,413.02	$0.00
37	29	$44,413.02	$0.00	$44,413.02	$2,664.78	$47,077.80	$0.00
38	30	$47,077.80	$0.00	$47,077.80	$2,824.67	$49,902.46	$0.00

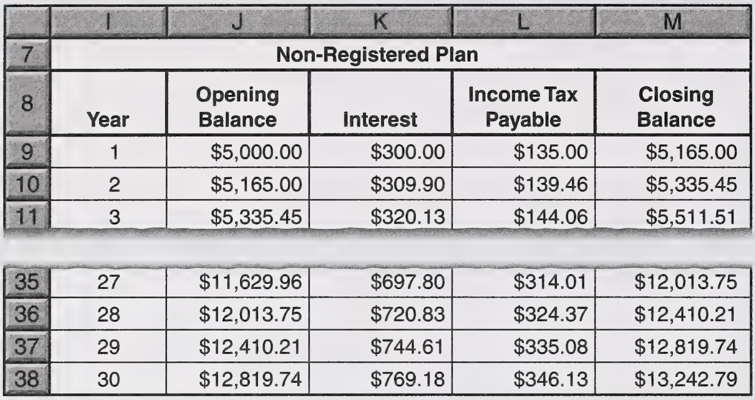
$$\begin{aligned}\text{Tax payable} &= \$49\,902.46 \times 0.45\\ &= \$22\,456.11\end{aligned}$$

$$\begin{aligned}\therefore\ \text{Net return} &= \$49\,902.46 - \$22\,456.11 - \$5000\\ &= \$22\,446.35\end{aligned}$$

Assume that the top tax rate is 45% so that this income of nearly $50 000 doesn't put the investor into a higher tax bracket.

Activity 2 (continued)

Non-Registered Plan



	I	J	K	L	M
7	Non-Registered Plan				
8	Year	Opening Balance	Interest	Income Tax Payable	Closing Balance
9	1	$5,000.00	$300.00	$135.00	$5,165.00
10	2	$5,165.00	$309.90	$139.46	$5,335.45
11	3	$5,335.45	$320.13	$144.06	$5,511.51
35	27	$11,629.96	$697.80	$314.01	$12,013.75
36	28	$12,013.75	$720.83	$324.37	$12,410.21
37	29	$12,410.21	$744.61	$335.08	$12,819.74
38	30	$12,819.74	$769.18	$346.13	$13,242.79

$$\begin{aligned}\text{Net return} &= \$13\,242.79 - \$5000\\ &= \$8242.79\end{aligned}$$

2.

$$\begin{aligned}\frac{\text{Net return}}{\text{Initial investment} \times \text{Number of years}} \times 100\% &= \frac{\$22\,446.35}{\$5000 \times 30} \times 100\%\\ &\doteq 14.96\%\end{aligned}$$

The RRSPs' average annual simple rate of return is about 14.96%.

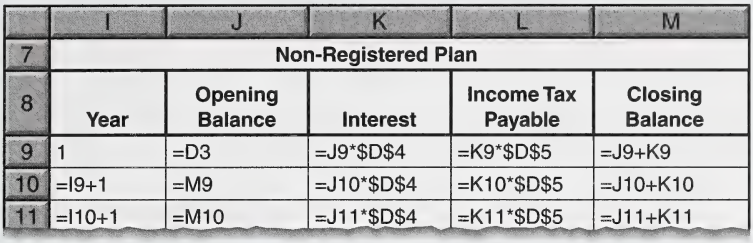
$$\begin{aligned}\frac{\text{Net return}}{\text{Initial investment} \times \text{Number of years}} \times 100\% &= \frac{\$8242.79}{\$5000 \times 30} \times 100\%\\ &\doteq 5.50\%\end{aligned}$$

The non-registered investments' average annual simple rate of return is about 5.50%.

Therefore, the RRSPs provide a better return, about 9.46% more.

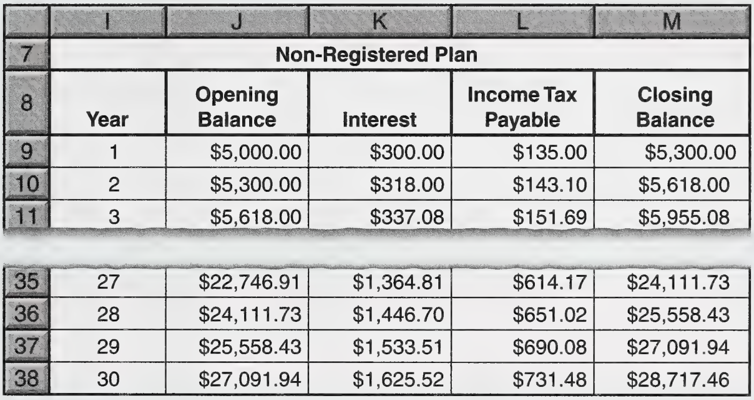
3. Textbook exercises 2, 3, and 4 of "Discussing the Ideas," p. 163

2. **a.** The spreadsheet would have to have the formulas for the Closing Balance changed so the Income Tax Payable is not subtracted from the Opening Balance. The new formulas are shown in the partial spreadsheet given.



	I	J	K	L	M
7	Non-Registered Plan				
8	Year	Opening Balance	Interest	Income Tax Payable	Closing Balance
9	1	=D3	=J9*$D$4	=K9*$D$5	=J9+K9
10	=I9+1	=M9	=J10*$D$4	=K10*$D$5	=J10+K10
11	=I10+1	=M10	=J11*$D$4	=K11*$D$5	=J11+K11

b. By not withdrawing from the investment to pay taxes, the net return on the investment would be greater because more interest would be earned due to the increased balance.



	I	J	K	L	M
7	Non-Registered Plan				
8	Year	Opening Balance	Interest	Income Tax Payable	Closing Balance
9	1	$5,000.00	$300.00	$135.00	$5,300.00
10	2	$5,300.00	$318.00	$143.10	$5,618.00
11	3	$5,618.00	$337.08	$151.69	$5,955.08
35	27	$22,746.91	$1,364.81	$614.17	$24,111.73
36	28	$24,111.73	$1,446.70	$651.02	$25,558.43
37	29	$25,558.43	$1,533.51	$690.08	$27,091.94
38	30	$27,091.94	$1,625.52	$731.48	$28,717.46

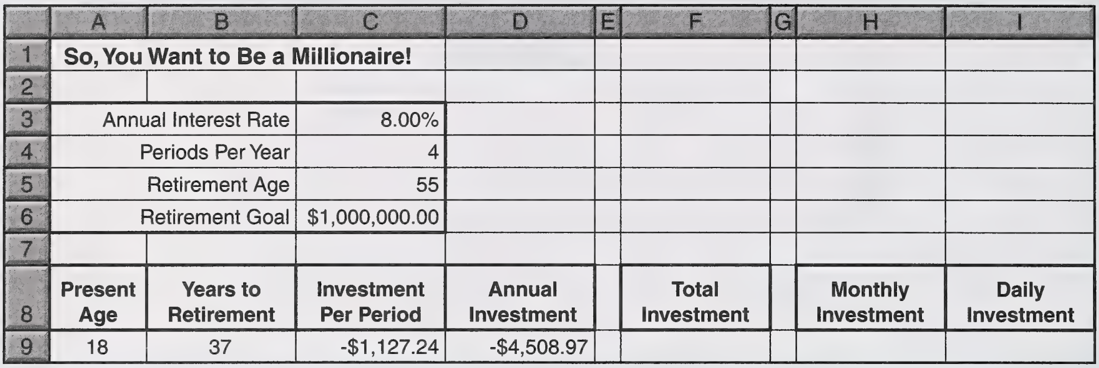
The closing balance after 30 years would be $28 717.46. This is an increase of $15 474.67.

3. The federal government is willing to defer tax on money invested in RRSPs so people will save for their retirement and become less reliant on government pensions. If retired people are self-sufficient with adequate income, they will require less in government subsidies. In fact, they will pay tax on their income.

4. In 2001, the maximum amount you can deposit into an RRSP is 18% of your earned income in 2000 or $13 500, whichever is less. The federal government places limits on RRSP contributions to limit the amount of tax that can be sheltered (deferred) in any given year.

4. **Textbook exercises 1 to 8 of "Investigation 2: So, You Want to Be a Millionaire!," pp. 161 and 162**

1. to 3. Format cell D3 as a percent and cell D6 as currency.

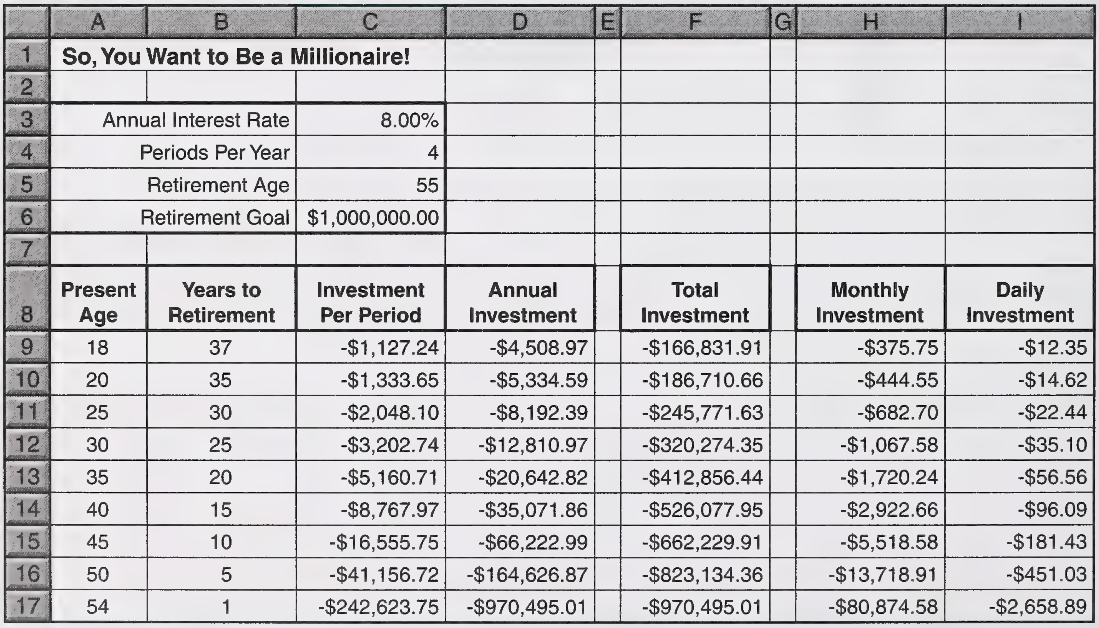


	A	B	C	D	E	F	G	H	I
1	**So, You Want to Be a Millionaire!**								
2									
3	Annual Interest Rate		8.00%						
4	Periods Per Year		4						
5	Retirement Age		55						
6	Retirement Goal		$1,000,000.00						
7									
8	**Present Age**	**Years to Retirement**	**Investment Per Period**	**Annual Investment**		**Total Investment**		**Monthly Investment**	**Daily Investment**
9	18	37	-$1,127.24	-$4,508.97					

4. The formulas are as follows:

- cell F9: =D9*B9
- cell H9: =D9/12
- cell I9: =D9/365

5.



	A	B	C	D	E	F	G	H	I
1	**So, You Want to Be a Millionaire!**								
2									
3	Annual Interest Rate		8.00%						
4	Periods Per Year		4						
5	Retirement Age		55						
6	Retirement Goal		$1,000,000.00						
7									
8	**Present Age**	**Years to Retirement**	**Investment Per Period**	**Annual Investment**		**Total Investment**		**Monthly Investment**	**Daily Investment**
9	18	37	-$1,127.24	-$4,508.97		-$166,831.91		-$375.75	-$12.35
10	20	35	-$1,333.65	-$5,334.59		-$186,710.66		-$444.55	-$14.62
11	25	30	-$2,048.10	-$8,192.39		-$245,771.63		-$682.70	-$22.44
12	30	25	-$3,202.74	-$12,810.97		-$320,274.35		-$1,067.58	-$35.10
13	35	20	-$5,160.71	-$20,642.82		-$412,856.44		-$1,720.24	-$56.56
14	40	15	-$8,767.97	-$35,071.86		-$526,077.95		-$2,922.66	-$96.09
15	45	10	-$16,555.75	-$66,222.99		-$662,229.91		-$5,518.58	-$181.43
16	50	5	-$41,156.72	-$164,626.87		-$823,134.36		-$13,718.91	-$451.03
17	54	1	-$242,623.75	-$970,495.01		-$970,495.01		-$80,874.58	-$2,658.89

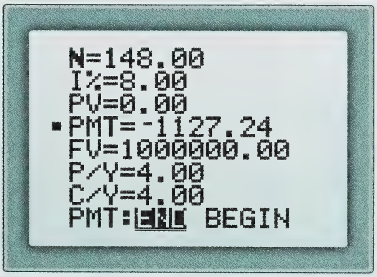
6. Save your spreadsheet as “Millionaire.”

7. The values in this table may differ from the spreadsheet due to rounding differences.

Present Age	Years to Retirement	Investment Per Period	Annual Investment	Total Investment	Monthly Investment	Daily Investment
18	37	$1127.24	$4508.96	$166 831.52	$375.75	$12.35
20	35	$1333.65	$5334.60	$186 711.00	$444.55	$14.62
25	30	$2048.10	$8192.40	$245.772.00	$682.70	$22.44
30	25	$3202.74	$12 810.96	$320 274.00	$1067.58	$35.10
35	20	$5160.71	$20 642.84	$412 856.80	$1720.24	$56.56
40	15	$8767.97	$35 071.88	$526 078.20	$2922.66	$96.09
45	10	$16 555.75	$66 223.00	$662 230.00	$5518.58	$181.43
50	5	$41 156.72	$164 626.88	$823 134.40	$13 718.91	$451.03
54	1	$242 623.75	$970 495.00	$970 495.00	$80 874.58	$2658.89

Use the TVM Solver to calculate the investment per period. For ease, adjust the mode of your graphing calculator to show the calculations rounded to 2 decimal places. The first two rows were calculated as follows:

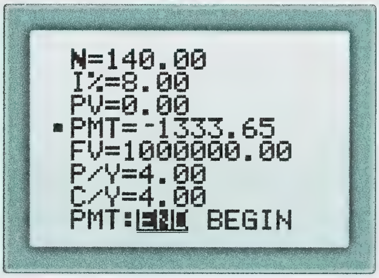
Age 18



```
 N=148.00
 I%=8.00
 PV=0.00
▪PMT=-1127.24
 FV=1000000.00
 P/Y=4.00
 C/Y=4.00
 PMT:END BEGIN
```

← = 37 × 4

Age 20



```
 N=140.00
 I%=8.00
 PV=0.00
▪PMT=-1333.65
 FV=1000000.00
 P/Y=4.00
 C/Y=4.00
 PMT:END BEGIN
```

← = 37 × 20

The remainder of table is calculated as follows:

- Annual investment = 4 × Investment per period
- Total investment = Years to retirement × Annual investment
- Monthly investment = Annual investment ÷ 12
- Daily investment = Annual investment ÷ 365

8. The goal of becoming a millionaire is likely no longer attainable at this interest rate after the age of 40. You would have to save almost $100 a day. A realistic age to begin to attain this dream may be about 21 to 25. Between ages 18 to 20, you may be attending school and can't afford to make payments; by age 30, the amount of the monthly investment may be too high.

5. Textbook exercises 1, 3, and 4 of "Exercises: Checking Your Skills," pp. 163 and 164

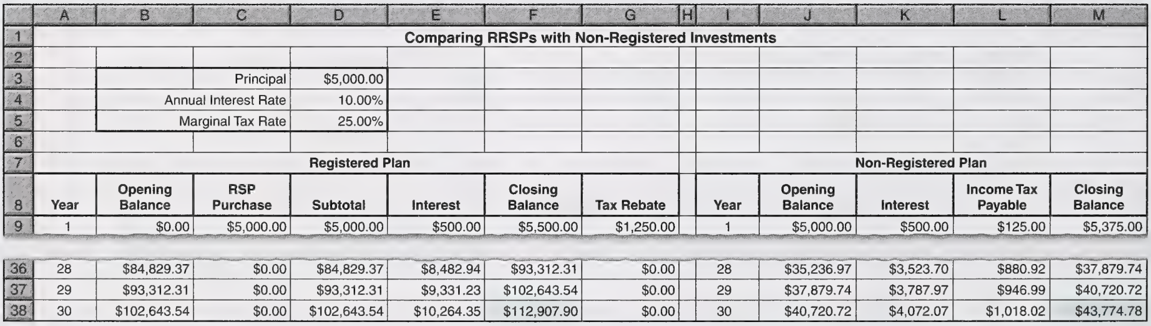
1.

Investing $5000 for 30 Years

Registered Plan					Non-Registered Plan	
Marginal Tax Rate	Closing Balance	Tax Paid	[illegible]	Average Annual Rate of Return	Average Annual Rate of Return	Closing Balance
25%	$112 907.90	$49 679.48	$58 228.42	38.82%	25.85%	$43 774.78
39%	$135 171.43	$59 475.43	$70 696.00	47.13%	16.36%	$29 541.43
44%	$145 411.69	$63 981.14	$76 430.55	50.95%	13.75%	$25 628.20

The calculations of the first row are as follows:

- The closing balance for both the registered plan and the non-registered plan are determined using the spreadsheet *Comparing RRSPs with Non-Registered Investments*.



	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Comparing RRSPs with Non-Registered Investments												
2													
3			Principal	$5,000.00									
4			Annual Interest Rate	10.00%									
5			Marginal Tax Rate	25.00%									
6													
7	Registered Plan								Non-Registered Plan				
8	Year	Opening Balance	RSP Purchase	Subtotal	Interest	Closing Balance	Tax Rebate		Year	Opening Balance	Interest	Income Tax Payable	Closing Balance
9	1	$0.00	$5,000.00	$5,000.00	$500.00	$5,500.00	$1,250.00		1	$5,000.00	$500.00	$125.00	$5,375.00
36	28	$84,829.37	$0.00	$84,829.37	$8,482.94	$93,312.31	$0.00		28	$35,236.97	$3,523.70	$880.92	$37,879.74
37	29	$93,312.31	$0.00	$93,312.31	$9,331.23	$102,643.54	$0.00		29	$37,879.74	$3,787.97	$946.99	$40,720.72
38	30	$102,643.54	$0.00	$102,643.54	$10,264.35	$112,907.90	$0.00		30	$40,720.72	$4,072.07	$1,018.02	$43,774.78

$$
\begin{aligned}
\text{Tax Paid} &= 44\% \text{ of Closing balance} \\
&= 0.44 \times \$112\,907.90 \\
&= \$49\,679.48
\end{aligned}
$$

Since this is a large amount of money, it will put the invester into the top tax bracket. If the money were taken out over several years, the tax paid could be much less.

- Net Return = Closing balance – Tax paid – $5000.00

$$\begin{aligned}&= \$112\,907.90 - \$49\,679.48 - \$5000.00\\ &= \$58\,228.42\end{aligned}$$

- Average Annual Rate of Return

$$\text{Registered Plan} = \frac{\text{Net return}}{\text{Initial investment} \times \text{Number of years}} \times 100\%$$

$$= \frac{\$58\,228.42}{\$5000.00 \times 30} \times 100\%$$

$$\doteq 38.82\%$$

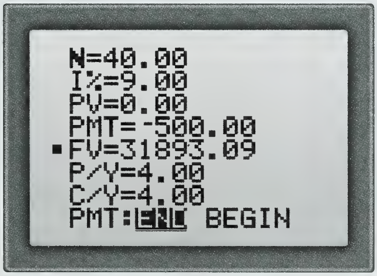
$$\text{Non-Registered Plan} = \frac{\text{Net return}}{\text{Initial investment} \times \text{Number of years}} \times 100\%$$

$$= \frac{\$43\,774.78 - \$5000.00}{\$5000.00 \times 30} \times 100\%$$

$$\doteq 25.85\%$$

The registered plan gives an average annual rate of return that is considerably larger than the non-registered plan. In the 25% bracket, the difference is 12.97%; in the 39% bracket, the difference is 30.77%; and in the 44% bracket, the difference is 37.20%.

3. a. Method 1: Using the TVM Solver



```
N=40.00
I%=9.00
PV=0.00
PMT=-500.00
▪FV=31893.09
P/Y=4.00
C/Y=4.00
PMT:END BEGIN
```

There are 40 quarterly payments over 10 years.

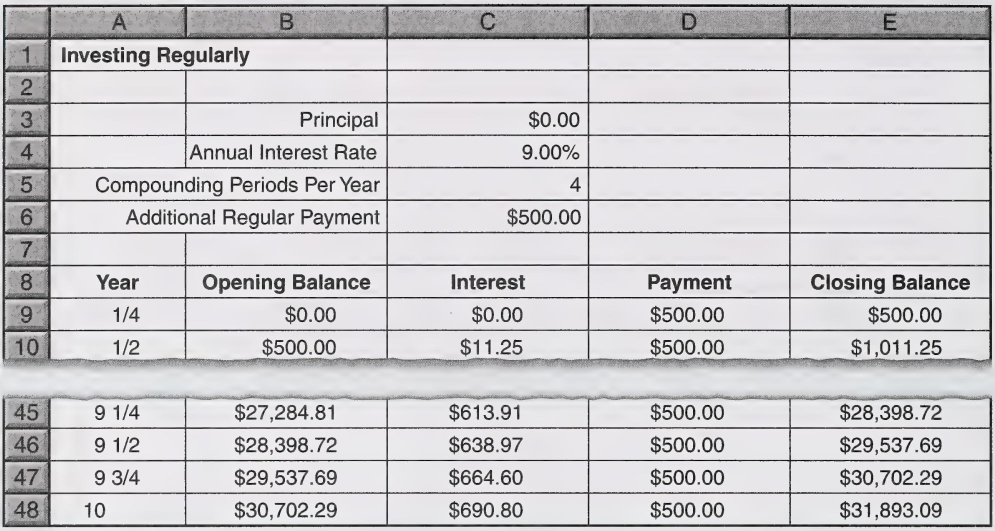
$500 is deposited every 3 months.

The payments are quarterly.

The interest is compounded quarterly.

You will have $31 893.09 after 10 years.

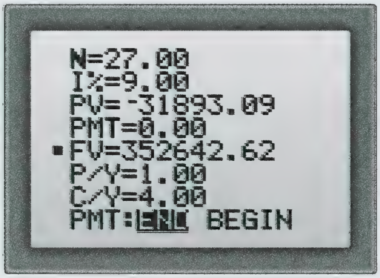
Method 2: Using a Spreadsheet



	A	B	C	D	E
1	**Investing Regularly**				
2					
3		Principal	$0.00		
4		Annual Interest Rate	9.00%		
5	Compounding Periods Per Year		4		
6	Additional Regular Payment		$500.00		
7					
8	**Year**	**Opening Balance**	**Interest**	**Payment**	**Closing Balance**
9	1/4	$0.00	$0.00	$500.00	$500.00
10	1/2	$500.00	$11.25	$500.00	$1,011.25
45	9 1/4	$27,284.81	$613.91	$500.00	$28,398.72
46	9 1/2	$28,398.72	$638.97	$500.00	$29,537.69
47	9 3/4	$29,537.69	$664.60	$500.00	$30,702.29
48	10	$30,702.29	$690.80	$500.00	$31,893.09

You will have $31 893.09 after 10 years.

b. Method 1: Using the TVM Solver



```
 N=27.00
 I%=9.00
 PV=-31893.09
 PMT=0.00
■FV=352642.62
 P/Y=1.00
 C/Y=4.00
 PMT:END BEGIN
```

← 55 – 28 = 27 years

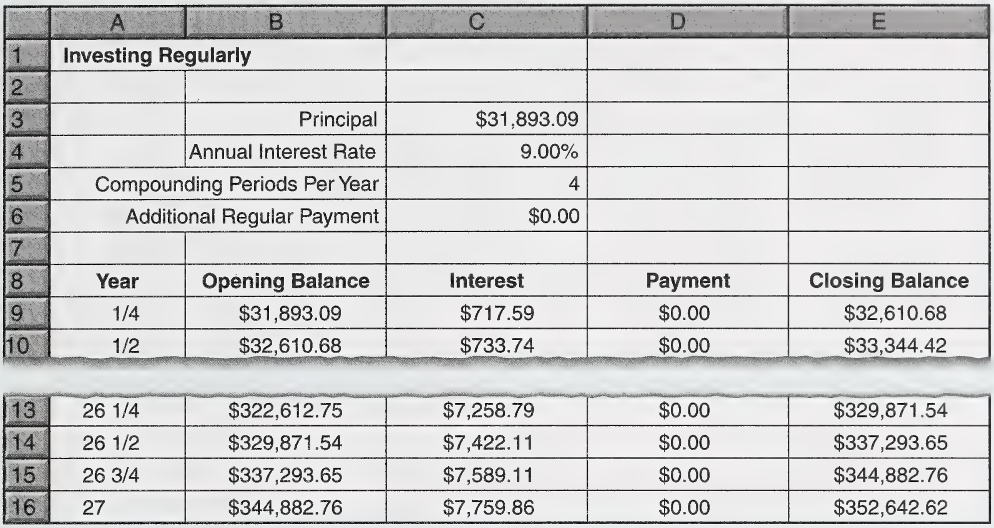
← This is the present value as determined in 3.a.

← Only the original deposit is made.

← The interest is still compounded quarterly.

You will have $352 642.62 at age 55.

Method 2: Using a Spreadsheet

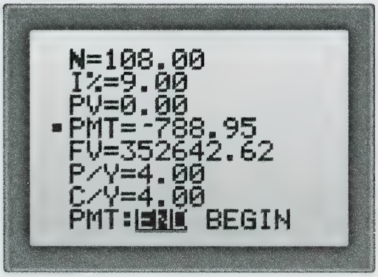


	A	B	C	D	E
1	**Investing Regularly**				
2					
3		Principal	$31,893.09		
4		Annual Interest Rate	9.00%		
5		Compounding Periods Per Year	4		
6		Additional Regular Payment	$0.00		
7					
8	**Year**	**Opening Balance**	**Interest**	**Payment**	**Closing Balance**
9	1/4	$31,893.09	$717.59	$0.00	$32,610.68
10	1/2	$32,610.68	$733.74	$0.00	$33,344.42
13	26 1/4	$322,612.75	$7,258.79	$0.00	$329,871.54
14	26 1/2	$329,871.54	$7,422.11	$0.00	$337,293.65
15	26 3/4	$337,293.65	$7,589.11	$0.00	$344,882.76
16	27	$344,882.76	$7,759.86	$0.00	$352,642.62

You will have $352 642.62 at age 55.

4. Use the TVM Solver to answer this exercise. The future value is $352 642.62.

a.



```
N=108.00
I%=9.00
PV=0.00
▪PMT=-788.95
FV=352642.62
P/Y=4.00
C/Y=4.00
PMT:END BEGIN
```

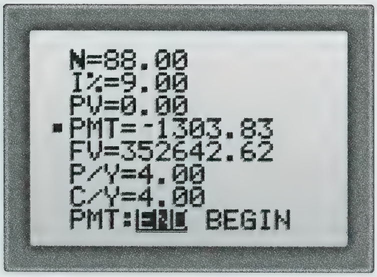
= 27 years × 4 payments per year

There are 4 payments per year.

The interest is compounded 4 times a year.

You must invest $788.95 quarterly.

b. If you start investing at 33, you will make $(55-33)\times 4 = 88$ deposits; so, $N = 88$.



```
N=88.00
I%=9.00
PV=0.00
▪PMT=-1303.83
FV=352642.62
P/Y=4.00
C/Y=4.00
PMT:END BEGIN
```

You must invest $1303.83 quarterly.

Activity 2 (continued)

6. Textbook exercise "Communicating the Ideas," p. 165

Answers will vary. A sample memo is given.

MEMORANDUM

TO: All Grade 12 Teachers
FROM: Financial Services
RE: Benefits of Investing

Financial Services is willing to visit the grade 12 students in your school to explain the benefits of investing in RRSPs and the importance of starting a plan at the earliest possible age.

As part of our presentation, we will demonstrate that the after-tax rate of return of an RRSP is higher than the rate of return for an equivalent investment outside an RRSP. We will use charts and spreadsheets to compare final balances in both types of investments and to show the power of compound interest, particularly when it is tax-sheltered in a RRSP. We will also demonstrate why it is important to start investing at the earliest possible age.

If you are interested, visits can be arranged through your principal.

Activity 3: Investment Portfolios

1. Textbook exercises 1 and 2 of "Discussing the Ideas," p. 170

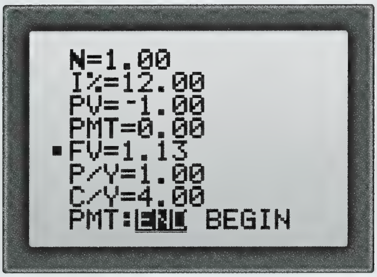
1. Most people in the 35 to 50 years range have disposable income that is higher than when they first started working; thus, they have more money to invest. People in this age range can also take more risks because it will still be 15 to 30 years before they retire; they can recover from downturns in the equity market or from bad investments.

2. A person may invest money in a savings account because it is a safe investment. It does earn some interest, however low, and the funds are readily accessible.

2. **Textbook exercises 2, 4, 5, and 6 of "Exercises: Checking Your Skills," pp. 171 and 172**

2. **Note:** Make sure the mode of your calculator is set to show calculations to 2 decimal places.

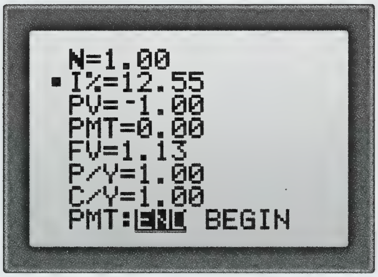
a. First, determine the amount at the end of 1 year of investing $1.00 at 12%, compounded quarterly.



```
 N=1.00
 I%=12.00
 PV=-1.00
 PMT=0.00
▪FV=1.13
 P/Y=1.00
 C/Y=4.00
 PMT:END BEGIN
```

← compounded quarterly

Now, determine the non-compounded rate that will give the amount just calculated.

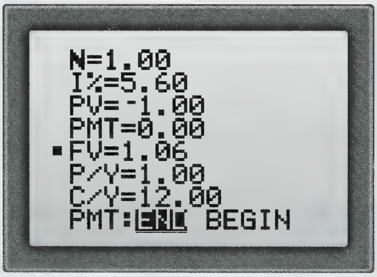


```
 N=1.00
▪I%=12.55
 PV=-1.00
 PMT=0.00
 FV=1.13
 P/Y=1.00
 C/Y=1.00
 PMT:END BEGIN
```

← compounded annually

The effective annual interest rate of 12%, compounded quarterly, is approximately 12.55%.

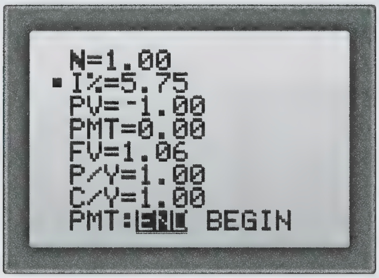
b. First, determine the amount at the end of 1 year of investing $1.00 at 5.6%, compounded monthly.



```
 N=1.00
 I%=5.60
 PV=-1.00
 PMT=0.00
▪FV=1.06
 P/Y=1.00
 C/Y=12.00
 PMT:END BEGIN
```

← compounded monthly

Now, determine the non-compounded rate that will give the amount just calculated.

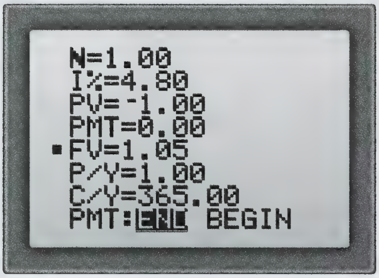


```
 N=1.00
▪I%=5.75
 PV=-1.00
 PMT=0.00
 FV=1.06
 P/Y=1.00
 C/Y=1.00
 PMT:END BEGIN
```

compounded annually

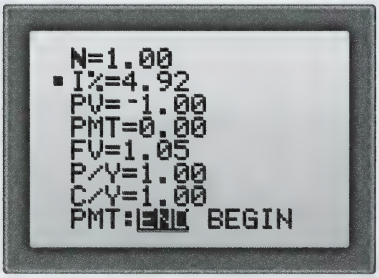
The effective annual interest rate of 5.6%, compounded monthly, is approximately 5.75%.

c. First, determine the amount at the end of 1 year of investing $1.00 at 4.8%, compounded daily.



```
 N=1.00
 I%=4.80
 PV=-1.00
 PMT=0.00
▪FV=1.05
 P/Y=1.00
 C/Y=365.00
 PMT:END BEGIN
```

Now, determine the non-compounded rate that will give the amount just calculated.



```
 N=1.00
▪I%=4.92
 PV=-1.00
 PMT=0.00
 FV=1.05
 P/Y=1.00
 C/Y=1.00
 PMT:END BEGIN
```

The effective annual interest rate of 4.8%, compounded daily, is approximately 4.92%.

4. a.

Investment	[illegible]	Rate of Return	[illegible]
Savings account	$1 800	2.5%	$45.00
Money market funds	$7 800	7.25%	$565.50
Canada Savings Bonds	$4 200	6.5%	$273.00
Term deposits	$5 600	7.0%	$392.00
Guaranteed investment certificates	$5 500	8.75%	$481.25
Stock in a computer company	$3 400	22.5%	$765.00
Mutual funds	$58 000	15.75%	$9 135.00
TOTALS	$86 300		$11 656.75

$$\begin{aligned}\text{Average annual rate of return} &= \frac{\text{Total return}}{\text{Total investment}} \times 100\% \\ &= \frac{\$11\,656.75}{\$86\,300.00} \times 100\% \\ &\doteq 13.51\%\end{aligned}$$

b. The average annual rate of return is a weighted average. It is not simply the average of the rates of return.

For example, suppose you invested $100, of which $1 was used to buy a cola. The cola was then sold to a friend for $2. The return on the $1 was 100%. Suppose the remaining $99 was placed in a sock and earned no income. The return on the $99 was 0%. Now, the return on the $100 is not $\frac{100\%+0\%}{2} = 50\%$; it is just $1 on $100, or 1%.

c. The electrician's savings, money market funds, and term deposits total $\$1800 + \$7800 + \$5600 = \$15\,200$. These are cash investments and represent $\frac{\$15\,200}{\$86\,300} \times 100\% \doteq 18\%$ of her total portfolio.

Her savings bonds and GICs total $\$4200 + \$5500 = \$9700$. These are fixed-income investments and represent $\frac{\$9700}{\$86\,300} \times 100\% \doteq 11\%$ of her total portfolio.

Her stock and mutual funds total $\$3400 + \$58\,000 = \$61\,400$. These are equity investments and represent $\frac{\$61\,400}{\$86\,300} \times 100\% \doteq 71\%$ of her total portfolio.

For someone her age, the recommended mix is 10% in cash, 40% in fixed income, and 50% in equities. Since she is over-invested in cash and equities, she should invest in some more fixed-income investments—probably in GICs because they are currently earning a higher rate of return than the bonds.

d. As determined in exercise 4.c., the electrician has about 18% in cash, about 11% in fixed income, and about 71% in equities. She should consider reducing her cash to 10% and her equities to 50% by moving them into fixed-income investments, such as GICs and bonds.

5.

Year	Opening Balance	Rate of Growth (%)	Closing Balance
1991	$10 000.00	8.2	$10 820.00
1992	$10 820.00	11.7	$12 085.94
1993	$12 085.94	14.6	$13 850.49
1994	$13 850.49	(2.9)	$13 448.83
1995	$13 448.83	33.1	$17 900.39

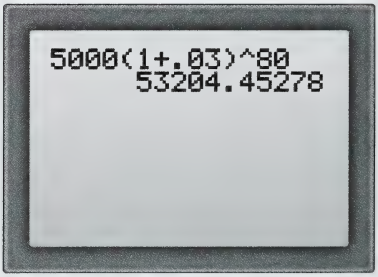
The total return on the $10 000 investment after 5 years is $\$17\,900.39 - \$10\,000.00 = \$7900.39$.

6. **a.** Use the formula for compound interest to determine the value of each GIC when she turns 65.

GIC Invested at Age 25

$$P = \$5000,\ i = \frac{0.06}{2} = 0.03, \text{ and } n = 40 \times 2 = 80$$

$$\begin{aligned} \therefore\ A &= P(1+i)^n \\ &= 5000(1+0.03)^{80} \\ &= \$53\,204.45 \end{aligned}$$

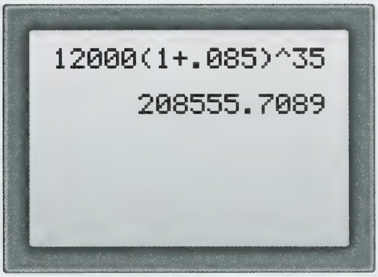


```
5000(1+.03)^80
        53204.45278
```

GIC Invested at Age 30

$P = \$12\,000$, $i = 0.085$, and $n = 35$

$$\begin{aligned}\therefore\ A &= P(1+i)^n \\ &= 12\,000(1+0.085)^{35} \\ &= \$208\,555.71\end{aligned}$$

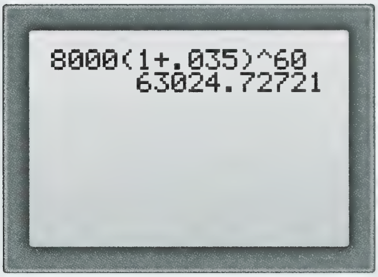


```
12000(1+.085)^35
             208555.7089
```

GIC Invested at Age 35

$$P = \$8000,\ i = \frac{0.07}{2} = 0.035, \text{ and } n = 30 \times 2 = 60$$

$$\begin{aligned}\therefore\ A &= P(1+i)^n \\ &= 8000(1+0.035)^{60} \\ &= \$63\,024.73\end{aligned}$$



```
8000(1+.035)^60
             63024.72721
```

$$\begin{aligned}\therefore\ \text{Total GICs} &= \$53\,204.45 + \$208\,555.71 + \$63\,024.73 \\ &= \$324\,784.89\end{aligned}$$

b. First, determine the value of the GICs when she turned 35.

GIC Invested at Age 25

$P = \$5000$, $i = 0.03$, and $n = 10 \times 2 = 20$

$$\begin{aligned}\therefore\ A &= P(1+i)^n \\ &= 5000(1+0.03)^{20} \\ &= \$9030.56\end{aligned}$$

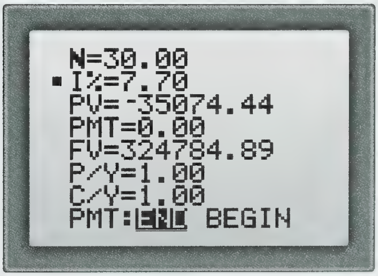
GIC Invested at Age 30

$P = \$12\,000$, $i = 0.085$, and $n = 5$

$$\begin{aligned}\therefore\ A &= P(1+i)^n \\ &= 12\,000(1+0.085)^{5} \\ &= \$18\,043.88\end{aligned}$$

$$\begin{aligned}\therefore\ \text{Total value of GICs at age 35} &= \$9030.56 + \$18\,043.88 + \$8000.00 \\ &= \$35\,074.44\end{aligned}$$

From exercise 6.a., the total value of her investment at age 65 will be $324 784.89. Use the TVM Solver to determine Samantha's average annual rate of return over the last 30 years.



```
 N=30.00
▪I%=7.70
 PV=-35074.44
 PMT=0.00
 FV=324784.89
 P/Y=1.00
 C/Y=1.00
 PMT:END BEGIN
```

Samantha's average annual rate of return for the last 30 years will be about 7.70%.

Activity 3 (continued)

3. Textbook exercise "Communicating the Ideas," p. 173

Answers will vary. A sample memo is given.

> **MEMORANDUM**
>
> FROM: Your Financial Advisor
> RE: A Long-Term Investment Strategy
>
> Congratulations on your graduation from college and your new job!
>
> I would like an opportunity to discuss your long-term financial strategy. As part of this discussion, we will explore a monthly investment plan that will include maximum RRSP contributions and investments held outside your RRSPs. For someone your age, our firm recommends 10% in cash investments, 40% in fixed-income investments, and 50% in equities.
>
> I look forward to meeting with you.

Activity 4: Leasing

1. Textbook exercises 1, 2, and 3 of "Investigation 1: Leases," pp. 174 and 175

1.

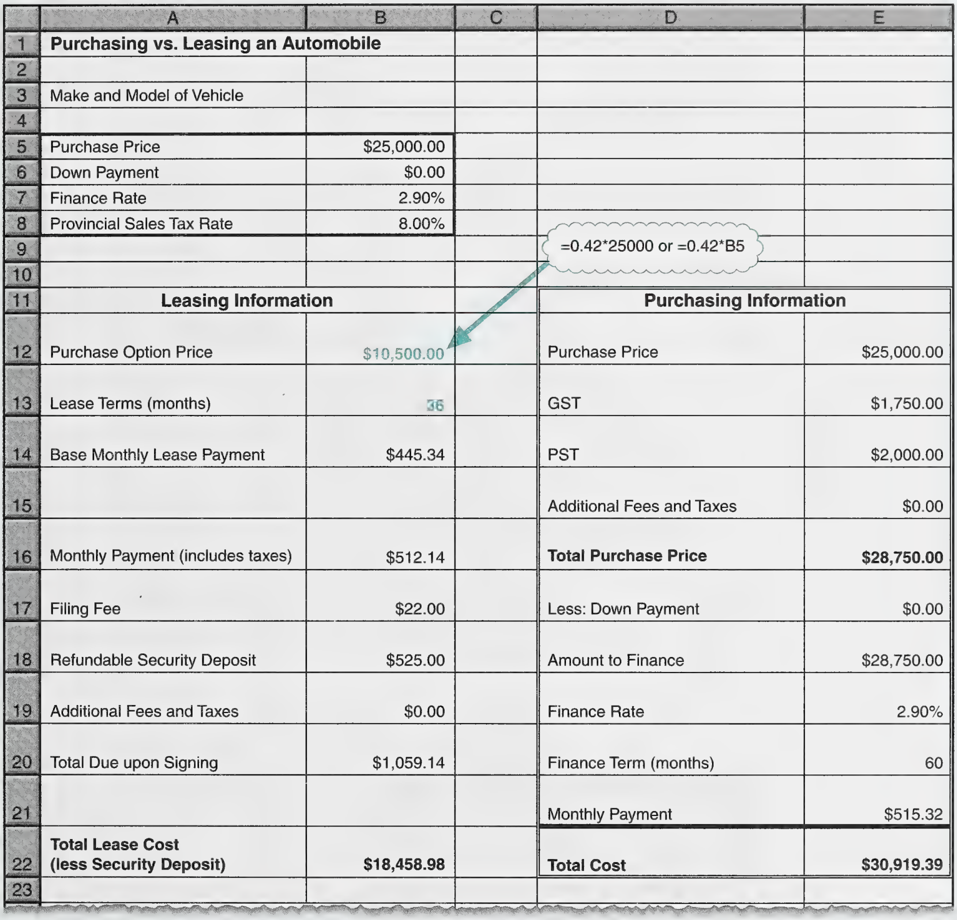
	Length of Lease (months)				
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Residual Value ($)	12 100.00	10 780.00	9240.00	7920.00	7040.00
Depreciation ($)	9900.00	11 220.00	12 760.00	14 080.00	14 960.00
Average Annual Cost (before interest and taxes) ($)	9900.00	5610.00	4253.33	3520.00	2992.00

2. A long-term lease is more cost-effective than a short-term lease because the vehicle depreciates the most in the first year. The cost for a one-year lease would be the highest.

The total cost if the car is purchased and financed over a 5-year period is $30 919.39. The total cost if the car is leased for 2 years and then purchased over a 3-year period is $30 636.40.

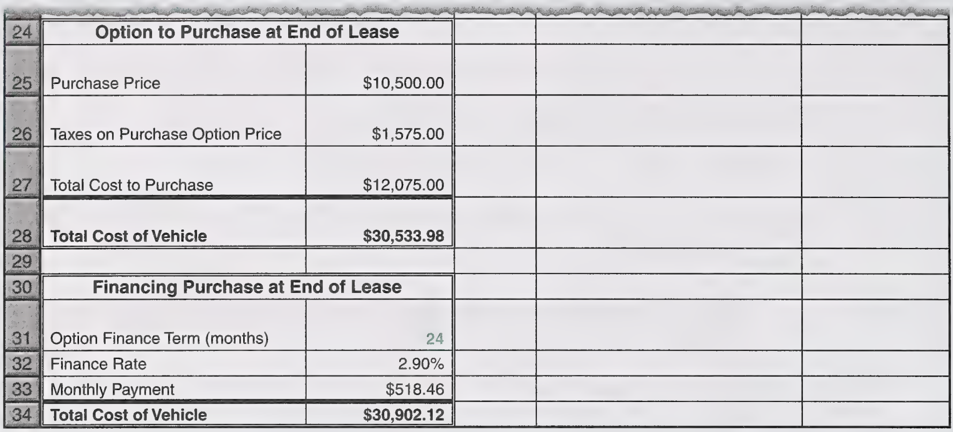
b. The monthly lease payment is $662.01, whereas the monthly payment to purchase is $515.32. The lease payment is larger because the term of the lease is only 24 months and 51% of the car's value is financed. If the vehicle is initially purchased, the financing is spread over 5 years.

c. Make the necessary changes to the spreadsheet. These are shown in colour on the following spreadsheet.



	A	B	C	D	E
1	**Purchasing vs. Leasing an Automobile**				
2					
3	Make and Model of Vehicle				
4					
5	Purchase Price	$25,000.00			
6	Down Payment	$0.00			
7	Finance Rate	2.90%			
8	Provincial Sales Tax Rate	8.00%			
9					
10					
11	**Leasing Information**			**Purchasing Information**	
12	Purchase Option Price	$10,500.00		Purchase Price	$25,000.00
13	Lease Terms (months)	36		GST	$1,750.00
14	Base Monthly Lease Payment	$445.34		PST	$2,000.00
15				Additional Fees and Taxes	$0.00
16	Monthly Payment (includes taxes)	$512.14		**Total Purchase Price**	**$28,750.00**
17	Filing Fee	$22.00		Less: Down Payment	$0.00
18	Refundable Security Deposit	$525.00		Amount to Finance	$28,750.00
19	Additional Fees and Taxes	$0.00		Finance Rate	2.90%
20	Total Due upon Signing	$1,059.14		Finance Term (months)	60
21				Monthly Payment	$515.32
22	**Total Lease Cost (less Security Deposit)**	**$18,458.98**		**Total Cost**	**$30,919.39**
23					

=0.42*25000 or =0.42*B5



24	**Option to Purchase at End of Lease**				
25	Purchase Price	$10,500.00			
26	Taxes on Purchase Option Price	$1,575.00			
27	Total Cost to Purchase	$12,075.00			
28	**Total Cost of Vehicle**	**$30,533.98**			
29					
30	**Financing Purchase at End of Lease**				
31	Option Finance Term (months)	24			
32	Finance Rate	2.90%			
33	Monthly Payment	$518.46			
34	**Total Cost of Vehicle**	**$30,902.12**			

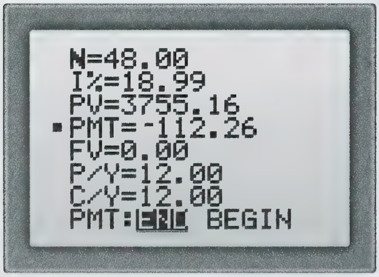
If the vehicle is leased over 3 years, the monthly lease payment is $512.14. The monthly payment for purchasing the vehicle over 5 years is $515.32.

d. The total cost if the car is purchased and financed over a 5-year period is $30 919.39. The total cost if the car is leased for 3 years and then purchased over a 2-year period is $30 902.12. Leasing is cheaper than purchasing because you are paying more of the cost (58%) of the car earlier at the same interest rate for purchasing the car over 5 years. The savings are due to savings in interest.

7. a. Determine the total cost of the computer.

$$\begin{aligned}\text{Total cost} &= \text{Purchase price} + \text{GST} + \text{PST}\\ &= \$3294.00 + \$3294.00(0.07 + 0.07)\\ &= \$3294.00 + \$461.16\\ &= \$3755.16\end{aligned}$$

Paid over 48 Months

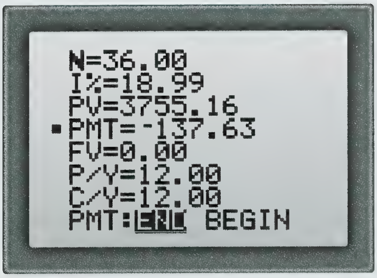


```
 N=48.00
 I%=18.99
 PV=3755.16
▪PMT=-112.26
 FV=0.00
 P/Y=12.00
 C/Y=12.00
 PMT:END BEGIN
```

$$\begin{aligned}\therefore\ \text{Total paid} &= \$112.26 \times 48\\ &= \$5388.48\end{aligned}$$

Remember: Since this is a purchase and not a lease, the payments are applied at the end of each month.

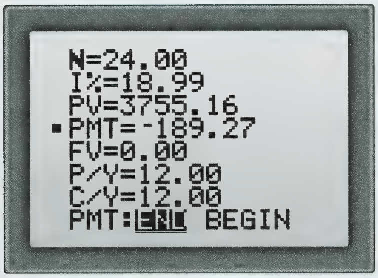
Paid over 36 Months



```
 N=36.00
 I%=18.99
 PV=3755.16
▪PMT=-137.63
 FV=0.00
 P/Y=12.00
 C/Y=12.00
 PMT:END BEGIN
```

$\therefore$ Total paid $= \$137.63 \times 36$
$= \$4954.68$

Paid over 24 Months



```
 N=24.00
 I%=18.99
 PV=3755.16
▪PMT=-189.27
 FV=0.00
 P/Y=12.00
 C/Y=12.00
 PMT:END BEGIN
```

$\therefore$ Total paid $= \$189.27 \times 24$
$= \$4542.48$

b.

Leasing Period	Monthly Rate	Total Paid
24 months	$182.88	$4389.12
36 months	$132.32	$4763.52
48 months	$106.92	$5132.16

c. **Comparison Between Buying and Leasing**

Period	Buying	Leasing
24 months	$4542.48	$4389.12
36 months	$4954.68	$4763.52
48 months	$5388.48	$5132.16

Leasing under these conditions is less expensive than financing a purchase.

4. a. Project Book exercise 1 of "Getting Started," p. 70

1.

Area of Flower Beds	$7\ \text{m}^2 + 8\ \text{m}^2 + 9\ \text{m}^2 = 24\ \text{m}^2$
Landscape Fabric Required	$24\ \text{m}^2$
Total Cost of Fabric	$24\ \text{m}^2 \times 0.50/\text{m}^2 = \$12.00$
Mulch Volume Required	$24\ \text{m}^2 \times 0.05\ \text{m} = 1.2\ \text{m}^3$
Total Cost of Mulch	$1.2\ \text{m}^3 \times \$10.00/\text{m}^3 = \$12.00$

b. Project Book exercise 2 of "Getting Started," p. 70

2. Answers may vary. A sample answer is given.

$$\begin{aligned}\text{Area of each tree} &= \pi r^2\\ &= \pi(0.5)^2 \longleftarrow \text{diameter} = 1.0\ \text{m}\\ &\doteq 0.7854\ \text{m}^2\end{aligned}$$

$$\begin{aligned}\text{Area of each shrub} &= \pi r^2\\ &= \pi(0.25)^2 \longleftarrow \text{diameter} = 0.5\ \text{m}\\ &\doteq 0.1963\ \text{m}^2\end{aligned}$$

Item	Total Number of Plants	Area (m^2)	Depth (m)	Approximate Volume (m^3)
Lawn	N/A	300	0.10	30.00
Trees (willows and maples)	7	5.50	1.00	5.50
Shrubs	8 Bamboo 14 Cedar 17 Rhododendron	7.66	0.50	3.83
Boxwood hedges	N/A	21.00 (42×0.5)	0.50	10.50
Bedding plants	N/A	24	0.50	12.00
Rock garden	N/A	50	1.00	50.00
			Total	111.83

The cost of the required topsoil is $112 \times \$15/\text{m}^3 \doteq \$1680$.

c. **Project Book exercise 3 of "Getting Started," p. 70**

3. Answers may vary. A sample answer is given.

   According to exercise 2, there is about 300 m^2 of lawn. If each bag covers 100 m^2, only 3 bags of lawn seed are required. Therefore, it will cost $3 \times \$15 = \$45$.

5. **Textbook exercise "Communicating the Ideas," p. 181**

Answers will vary. Your pamphlet should include the following:

- information about lease payments and that they are based on the difference between the purchase price of the car and the option-to-purchase price of the vehicle at the end of the lease
- conditions of the lease (such as additional costs for driving the vehicle over the maximum distances), service charges, and insurance
- advantages to the lessee, such as lower monthly payments, being able to drive a more-expensive vehicle for the same money required to purchase a less-expensive car, driving a new vehicle more often, no resale hassles at the end of the lease, a guaranteed purchase price at the end of the lease, and possible tax advantages for business (**Note:** It is a good idea to include a cost comparison between purchasing and leasing a particular car to illustrate the advantages.)

Activity 5: Mortgages

1. **Textbook exercises 1 to 5 of "Investigation 1: Which House Can You Afford?," pp. 184 and 185**

1. $$\begin{aligned}\text{Lars's weekly earnings} &= \$7.50/\text{h}(35\,\text{h}) + \$425.00\\ &= \$262.50 + \$425.00\\ &= \$687.50\end{aligned}$$

   $$\begin{aligned}\text{Freya's monthly earnings} &= \$1050.00 \times 2\\ &= \$2100.00\end{aligned}$$

   $$\begin{aligned}\therefore \text{Lars's monthly earnings} &= \$687.50 \times 4\\ &= \$2750.00\end{aligned}$$

   The gross monthly income for Lars and Freya is $\$2750.00 + \$2100.00 = \$4850.00$.

2. $$\begin{aligned}32\% \text{ of } \$4850.00 &= 0.32 \times \$4850.00\\ &= \$1552.00\end{aligned}$$

   The maximum monthly house payment that should be considered is \$1552.

3. 40% of $\$4850.00 = 0.48 \times \$4850.00$
$$= \$1940.00$$

$$\therefore \text{ Maximum monthly house payment} = \$1940.00 - (\$335.00 + \$250.00)$$
$$= \$1940.00 - \$585.00$$
$$= \$1355.00$$

4. The lesser of the answers for exercises 2 and 3 is $1355.00.

   Lars and Freya will pay $\$75.00 + \$220.00 = \$295.00$ a month in heating and taxes.

   $\$1355.00 - \$295.00 = \$1060.00$

   The maximum amount they may spend on mortgage payments is $1060.

5. According to the table, every $1000 of their mortgage costs $8.052 27 a month in interest and principal. If they can spend $1060.00 a month on mortgage payments, the maximum amount the bank will lend them is obtained from the following formula:

$$\text{Mortgage amount} = \frac{\text{Payment} \times \$1000}{\text{Cost per } \$1000}$$
$$= \frac{\$1060.00 \times \$1000}{\$8.052\ 27}$$
$$= \$131\ 639.90$$

   The maximum mortgage the bank will lend them is about $131 639.90.

2. a. **Textbook exercises 1, 2, and 3 of "Discussing the Ideas," p. 187**

   1. There is a range for the maximum amount of mortgage allowed based on family income because individual circumstances differ. Some families may have several children to look after, some may live in an area of the country where other expenses may be high, or some may have their income not keep up with inflation.

   2. Rather than basing the mortgage allowable on gross family income, a better method might be to base the formula on after-tax income.

   3. The balance of a mortgage shrinks slowly during the first few years because the majority of the mortgage payment goes toward the interest rather than the principal.

3. **a.** Lease payments and interest are calculated monthly, so $N = 24$.

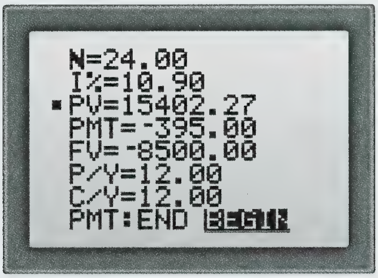
The annual rate is 10.9%, so $I\% = 10.9$.

The monthly payments are \$395, so $PMT = -395$.

The residual value is \$8500, so $FV = -8500$. This value is negative because it is an amount you would pay to purchase the car.

P/Y and C/Y = 12.

Payments are made at the beginning of each month, so select "BEGIN."



```
 N=24.00
 I%=10.90
■PV=15402.27
 PMT=-395.00
 FV=-8500.00
 P/Y=12.00
 C/Y=12.00
 PMT:END BEGIN
```

The present value of the lease is \$15 402.27.

b. The total price charged for the car is the sum of the present value of the lease and the down payment.

$$\begin{aligned} \therefore \text{ Total price} &= \$15\,402.27 + \$2350.00 \\ &= \$17\,752.27 \end{aligned}$$

c. The car is leased for 24 months at \$395 per month, with a down payment of \$2350.

$$\begin{aligned} \therefore \text{ Total paid at lease end} &= \$395 \times 24 + \$2350 \\ &= \$11\,830 \end{aligned}$$

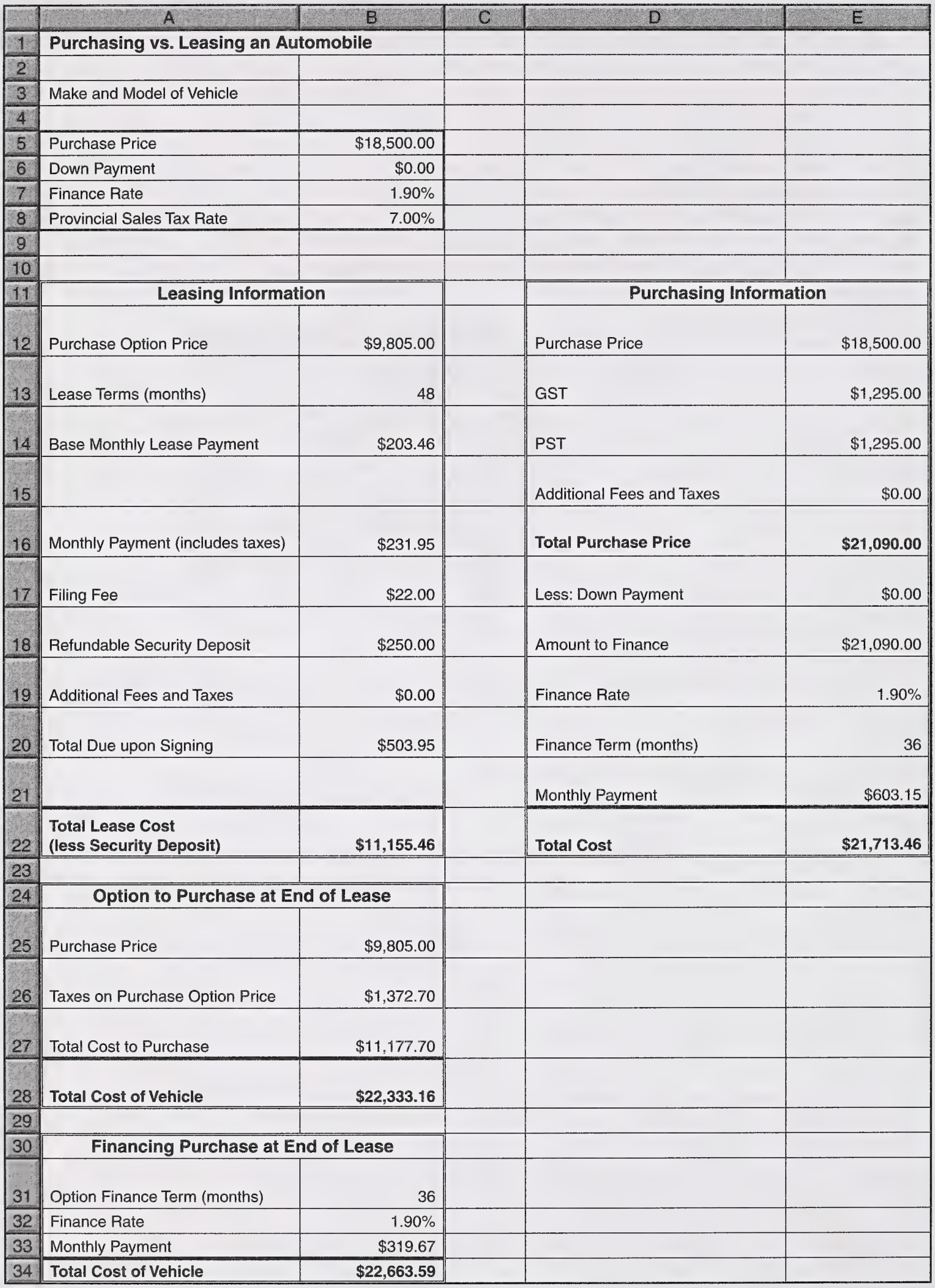
d. $\$11\,830 + \$8500 = \$20\,330$

If the car is purchased at the end of the lease for the residual value, \$8500, the total paid for the car without taxes is \$20 330.

Activity 4 (continued)

2. Textbook exercises 1 to 4 of "Investigation 2: To Lease or to Buy?," pp. 175 to 177

1.



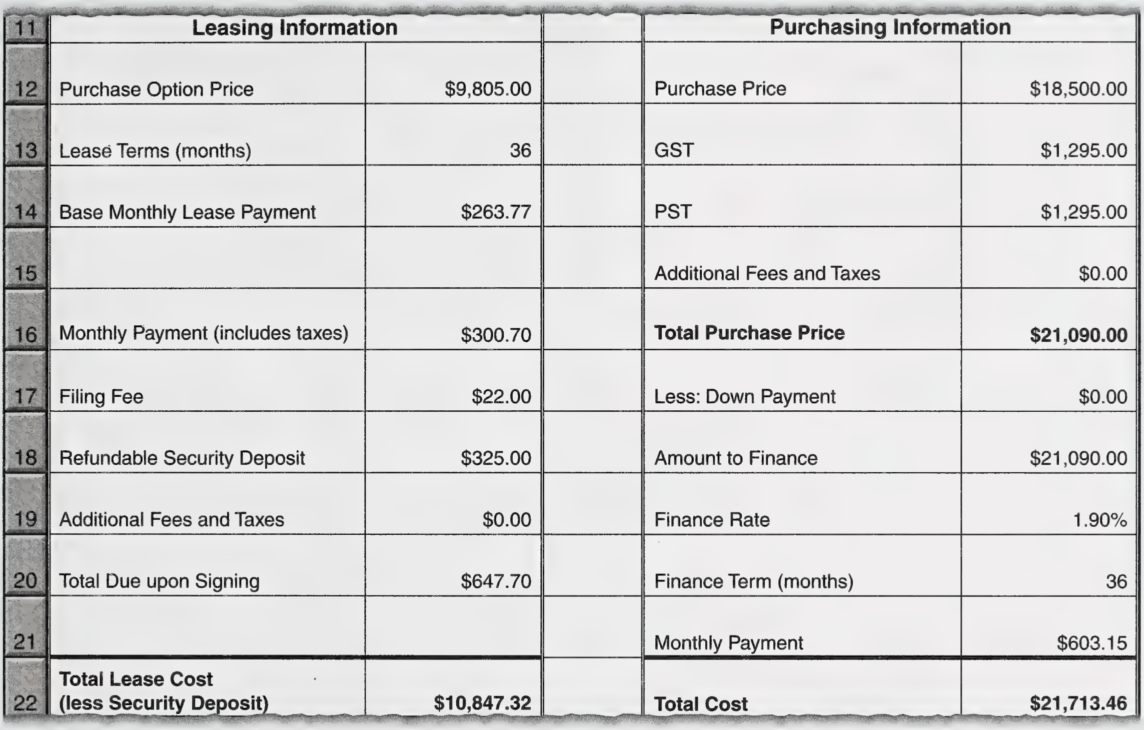
	A	B	C	D	E
1	**Purchasing vs. Leasing an Automobile**				
2					
3	Make and Model of Vehicle				
4					
5	Purchase Price	$18,500.00			
6	Down Payment	$0.00			
7	Finance Rate	1.90%			
8	Provincial Sales Tax Rate	7.00%			
9					
10					
11	**Leasing Information**			**Purchasing Information**	
12	Purchase Option Price	$9,805.00		Purchase Price	$18,500.00
13	Lease Terms (months)	48		GST	$1,295.00
14	Base Monthly Lease Payment	$203.46		PST	$1,295.00
15				Additional Fees and Taxes	$0.00
16	Monthly Payment (includes taxes)	$231.95		**Total Purchase Price**	**$21,090.00**
17	Filing Fee	$22.00		Less: Down Payment	$0.00
18	Refundable Security Deposit	$250.00		Amount to Finance	$21,090.00
19	Additional Fees and Taxes	$0.00		Finance Rate	1.90%
20	Total Due upon Signing	$503.95		Finance Term (months)	36
21				Monthly Payment	$603.15
22	**Total Lease Cost (less Security Deposit)**	**$11,155.46**		**Total Cost**	**$21,713.46**
23					
24	**Option to Purchase at End of Lease**				
25	Purchase Price	$9,805.00			
26	Taxes on Purchase Option Price	$1,372.70			
27	Total Cost to Purchase	$11,177.70			
28	**Total Cost of Vehicle**	**$22,333.16**			
29					
30	**Financing Purchase at End of Lease**				
31	Option Finance Term (months)	36			
32	Finance Rate	1.90%			
33	Monthly Payment	$319.67			
34	**Total Cost of Vehicle**	**$22,663.59**			

2. **a.** The total leasing cost (cell B22) is $11 155.46.

   b. The total cost of the vehicle if the option price is financed at the end of the lease (cell B34) is $22 663.59. If the vehicle is purchased at the end of the lease without financing, the purchase price is $22 333.16 (cell B28).

   c. The total cost to purchase (cell E22) is $21 713.46.

3. **a.**

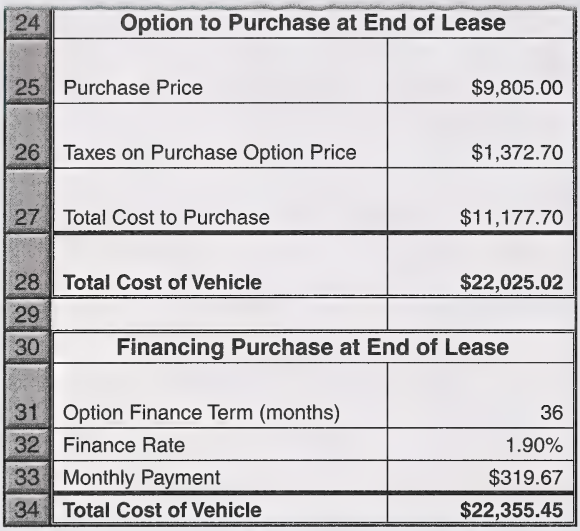


11	**Leasing Information**			**Purchasing Information**	
12	Purchase Option Price	$9,805.00		Purchase Price	$18,500.00
13	Lease Terms (months)	36		GST	$1,295.00
14	Base Monthly Lease Payment	$263.77		PST	$1,295.00
15				Additional Fees and Taxes	$0.00
16	Monthly Payment (includes taxes)	$300.70		**Total Purchase Price**	**$21,090.00**
17	Filing Fee	$22.00		Less: Down Payment	$0.00
18	Refundable Security Deposit	$325.00		Amount to Finance	$21,090.00
19	Additional Fees and Taxes	$0.00		Finance Rate	1.90%
20	Total Due upon Signing	$647.70		Finance Term (months)	36
21				Monthly Payment	$603.15
22	**Total Lease Cost (less Security Deposit)**	**$10,847.32**		**Total Cost**	**$21,713.46**

$\$603.15-\$300.70=\$302.45$

The difference between the monthly cost to purchase and the monthly cost to lease is $302.45.

b.



24	**Option to Purchase at End of Lease**	
25	Purchase Price	$9,805.00
26	Taxes on Purchase Option Price	$1,372.70
27	Total Cost to Purchase	$11,177.70
28	**Total Cost of Vehicle**	**$22,025.02**
29		
30	**Financing Purchase at End of Lease**	
31	Option Finance Term (months)	36
32	Finance Rate	1.90%
33	Monthly Payment	$319.67
34	**Total Cost of Vehicle**	**$22,355.45**

$22 355.45 – $21 713.46 = $641.99

The difference between the purchase costs through financing at the end of the lease and the purchase cost if you purchase the car through financing to begin with is $641.99. **Note:** The textbook answer used the total cost of the vehicle from line 28 rather than from line 34. Line 34 includes the finance charges on the option to purchase.

4. **a.** Cost of financing = Total cost – Total purchase price

$$\begin{aligned} &= \$21\,713.46 - \$21\,090.00 \\ &= \$623.46 \end{aligned}$$

The cost of financing if you purchase the vehicle is $623.46.

b. Cost of financing = Total cost – Total purchase price

$$\begin{aligned} &= \$22\,355.45 - \$21\,090.00 \\ &= \$1265.45 \end{aligned}$$

The cost of financing if you lease the vehicle and then purchase it at the end of the lease is $1265.45.

3. **a.** **Textbook exercises 1 to 4 of "Discussing the Ideas," p. 178**

1. People lease vehicles to reduce their monthly payments, drive a new vehicle at the end of the lease, avoid the hassle of selling a used vehicle, or use the lease payments as a business expense.

2. Consumers may lease computer equipment, apartments, furniture, and construction equipment to name a few.

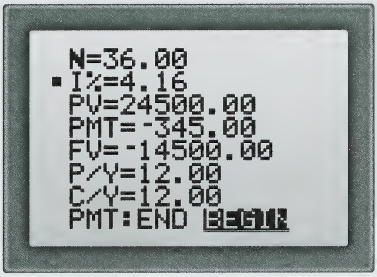
3. Businesses may lease vehicles, business locations, computer equipment, construction equipment, and furniture to name a few.

4. Some of the advantages of leasing are lower payments, the return of leased items at the end of the lease, the avoidance of having to sell used items, and tax advantages for businesses.

   The disadvantages of leasing are you do not own the items at the end of the lease, your leasing costs are ongoing, and you end up paying more for leasing over time.

b. Textbook exercises 1, 2, and 7 of "Exercises: Checking Your Skills," pp. 178 to 180

1. **a.** Using the TVM Solver, enter the given values and solve for I%.



```
 N=36.00
▪I%=4.16
 PV=24500.00
 PMT=-345.00
 FV=-14500.00
 P/Y=12.00
 C/Y=12.00
 PMT:END BEGIN
```

Remember: The present value (PV) is positive because you receive this amount. The future value (FV) is negative because this is the amount you owe at the end of the lease.

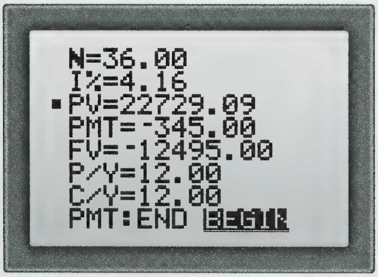
The finance rate is 4.16%.

b. From the table on page 174 of the textbook, the residual value of a luxury car after 36 months is 51% of its purchase price.

$$\begin{aligned} \therefore \text{ Residual value} &= 51\% \text{ of } \$24\ 500 \\ &= 0.51 \times \$24\ 500 \\ &= \$12\ 495 \end{aligned}$$

The residual value from the table is $\$14\ 500 - \$12\ 495 = \$2005$ less than the purchase option price.

c. Use the TVM Solver to find the present value of the lease. The present value is the purchase price reduced by the down payment.



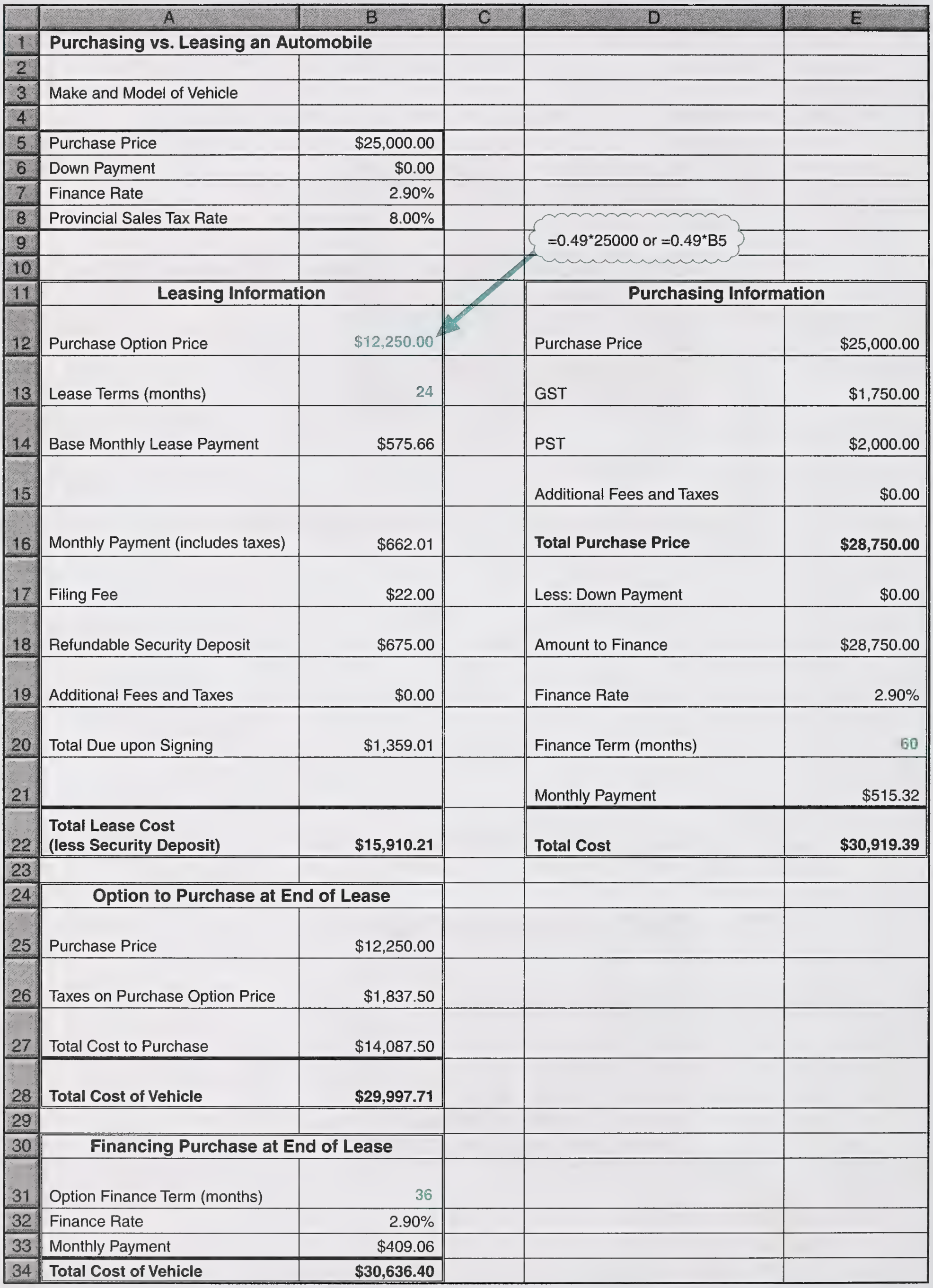
```
 N=36.00
 I%=4.16
▪PV=22729.09
 PMT=-345.00
 FV=-12495.00
 P/Y=12.00
 C/Y=12.00
 PMT:END BEGIN
```

Remember: You must re-enter the interest rate as 4.16%; otherwise, the calculator will use the unrounded value (4.158 597 101) calculated in exercise 1.a.

The present value is $22 729.09.

Therefore, the down payment was $\$24\ 500.00 - \$22\ 729.09 = \$1770.91$.

2. **a.** Enter the values from the problem. These are shown in colour on the following spreadsheet.



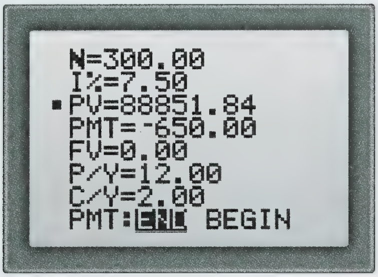
	A	B	C	D	E
1	**Purchasing vs. Leasing an Automobile**				
2					
3	Make and Model of Vehicle				
4					
5	Purchase Price	$25,000.00			
6	Down Payment	$0.00			
7	Finance Rate	2.90%			
8	Provincial Sales Tax Rate	8.00%			
9					
10					
11	**Leasing Information**			**Purchasing Information**	
12	Purchase Option Price	$12,250.00		Purchase Price	$25,000.00
13	Lease Terms (months)	24		GST	$1,750.00
14	Base Monthly Lease Payment	$575.66		PST	$2,000.00
15				Additional Fees and Taxes	$0.00
16	Monthly Payment (includes taxes)	$662.01		**Total Purchase Price**	**$28,750.00**
17	Filing Fee	$22.00		Less: Down Payment	$0.00
18	Refundable Security Deposit	$675.00		Amount to Finance	$28,750.00
19	Additional Fees and Taxes	$0.00		Finance Rate	2.90%
20	Total Due upon Signing	$1,359.01		Finance Term (months)	60
21				Monthly Payment	$515.32
22	**Total Lease Cost (less Security Deposit)**	**$15,910.21**		**Total Cost**	**$30,919.39**
23					
24	**Option to Purchase at End of Lease**				
25	Purchase Price	$12,250.00			
26	Taxes on Purchase Option Price	$1,837.50			
27	Total Cost to Purchase	$14,087.50			
28	**Total Cost of Vehicle**	**$29,997.71**			
29					
30	**Financing Purchase at End of Lease**				
31	Option Finance Term (months)	36			
32	Finance Rate	2.90%			
33	Monthly Payment	$409.06			
34	**Total Cost of Vehicle**	**$30,636.40**			

=0.49*25000 or =0.49*B5

b. Textbook exercises 1.a., 1.b., 2, 3.a., 3.b., 3.c., 5.a., 6, and 7 of "Exercises: Checking Your Skills," pp. 187 to 189

1. Use the TVM Solver.

   a. Enter the values, and solve for PV.

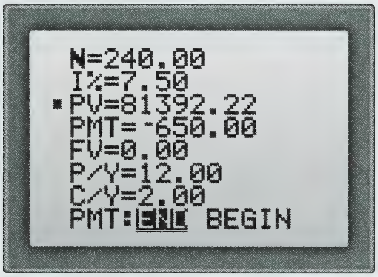


```
 N=300.00
 I%=7.50
▪PV=88851.84
 PMT=-650.00
 FV=0.00
 P/Y=12.00
 C/Y=2.00
 PMT:END BEGIN
```

← 25 years × 12 payments/year = 300 payments

Anita can afford a $88 851.84 mortgage amortized over 25 years.

   b. Enter the values, and solve for PV.



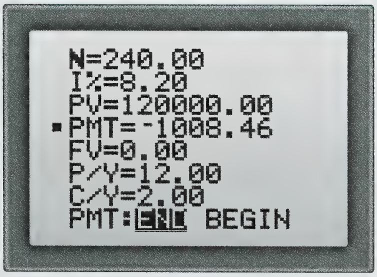
```
 N=240.00
 I%=7.50
▪PV=81392.22
 PMT=-650.00
 FV=0.00
 P/Y=12.00
 C/Y=2.00
 PMT:END BEGIN
```

← = 20 × 12

Anita can afford a $81 392.22 mortgage amortized over 20 years.

2. Use the TVM Solver on your graphing calculator to determine the monthly mortgage payments.

Mortgage A: $120 000 at 8.2% over 20 years



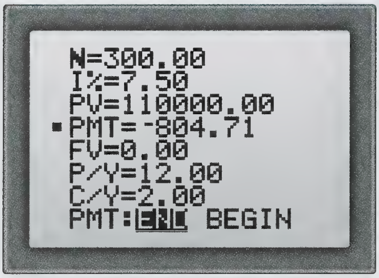
```
 N=240.00
 I%=8.20
 PV=120000.00
▪PMT=-1008.46
 FV=0.00
 P/Y=12.00
 C/Y=2.00
 PMT:END BEGIN
```

← = 20 × 12

The monthly mortgage payment is $1008.46.

$$\therefore \text{ Total cost of mortgage} = \$1008.46 \times 240$$
$$= \$242\ 030.40$$

Mortgage B: $110 000 at 7.5% over 25 years



```
 N=300.00
 I%=7.50
 PV=110000.00
▪PMT=-804.71
 FV=0.00
 P/Y=12.00
 C/Y=2.00
 PMT:END BEGIN
```

← = 25 × 12

The monthly mortgage payment is $804.71.

$$\therefore \text{ Total cost of mortgage} = \$804.71 \times 300$$
$$= \$241\ 413.00$$

$$\$242\ 030.40 - \$241\ 413.00 = \$617.40$$

Mortgage B, the $110 000 mortgage, would cost $617.40 less.

3. **a.** Grant's total monthly payments for his house and other obligations should not exceed 40% of his gross monthly income.

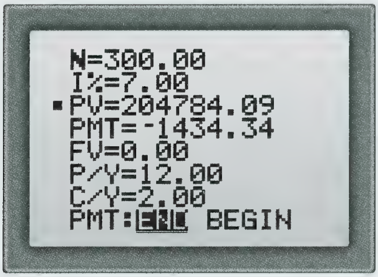
$$\text{Gross monthly income} = \frac{\$65\ 000}{12}$$
$$= \$5416.67$$

$$\therefore \ 40\% \text{ of monthly income} = 0.40 \times \$5416.67$$
$$= \$2166.67$$

Grant pays $339/month on a personal loan, $333.33/month on property taxes $(\$4000 \div 12)$, and $60/month for heat.

$$\therefore \text{ Maximum mortgage payment} = \$2166.67 - (\$339.00 + \$333.33 + \$60.00)$$
$$= \$2166.67 - \$732.33$$
$$= \$1434.34 \text{ /month}$$

b. Use the TVM Solver. Enter the known values, and solve for PV. **Note:** You could also use the table method in exercise 5 on page 185 of the textbook.

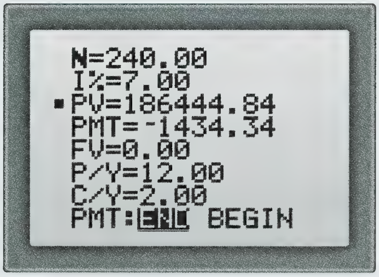


```
 N=300.00
 I%=7.00
▪PV=204784.09
 PMT=-1434.34
 FV=0.00
 P/Y=12.00
 C/Y=2.00
 PMT:END BEGIN
```

The maximum mortgage for which Grant qualifies is $204 784.09.

c. Use your TVM Solver to determine the maximum mortgage available for 20 years, 15 years, and 10 years.

20-Year Mortgage

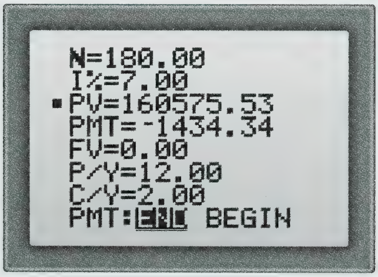


```
 N=240.00
 I%=7.00
▪PV=186444.84
 PMT=-1434.34
 FV=0.00
 P/Y=12.00
 C/Y=2.00
 PMT:END BEGIN
```

← = 20 × 12

Grant could qualify for a $186 444.84 mortgage amortized over 20 years.

15-Year Mortgage

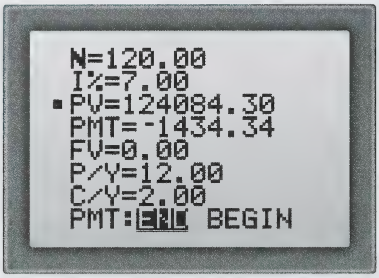


```
 N=180.00
 I%=7.00
▪PV=160575.53
 PMT=-1434.34
 FV=0.00
 P/Y=12.00
 C/Y=2.00
 PMT:END BEGIN
```

← = 15 × 12

Grant could qualify for a $160 575.53 mortgage amortized over 15 years.

10-Year Mortgage



```
N=120.00
I%=7.00
▪PV=124084.30
PMT=-1434.34
FV=0.00
P/Y=12.00
C/Y=2.00
PMT:END BEGIN
```

= 10 × 12

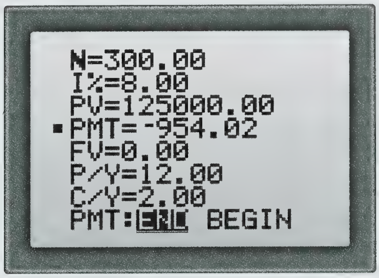
Grant could qualify for a $124 084.30 mortgage amortized over 10 years.

5. **a.** Use the TVM Solver to determine the monthly payments at 6.50%.

Amortization Period (years)	Monthly Payments	Total Payments
15	$866.37	$155 946.60
20	$740.50	$177 720.00
25	$669.82	$200 946.00

Comparing Amortization Periods (years)	Total Savings
20 vs. 25	$23 226.00
15 vs. 25	$44 999.40
15 vs. 20	$21 773.40

6. **a.** First, use the TVM Solver to determine their monthly payments.

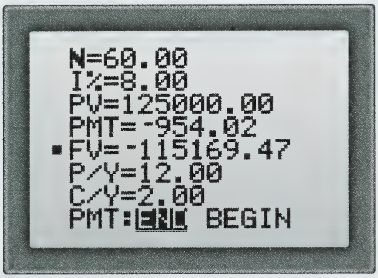


```
 N=300.00
 I%=8.00
 PV=125000.00
▪PMT=-954.02
 FV=0.00
 P/Y=12.00
 C/Y=2.00
 PMT:END BEGIN
```

= 25 × 12

Drew and Janine's monthly payment is $954.02.

Next, use the TVM Solver to determine the amount owing on the mortgage after 5 years.



```
 N=60.00
 I%=8.00
 PV=125000.00
 PMT=-954.02
▪FV=-115169.47
 P/Y=12.00
 C/Y=2.00
 PMT:END BEGIN
```

= 5 × 12

Note: Make sure you re-enter the payment; otherwise, the TVM Solver will use the unrounded amount of the payment.

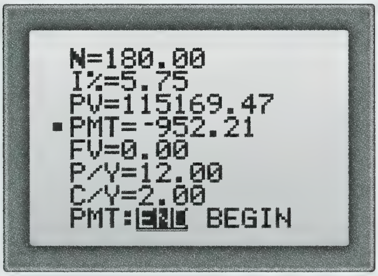
The amount owing after 5 years is $115 169.47.

$$\text{Total paid} = \$954.02 \times 60 = \$57\,241.20$$

$$\text{Difference in principal} = \$125\,000.00 - \$115\,169.47 = \$9830.53$$

$$\therefore \text{ Total interest paid} = \$57\,241.20 - \$9830.53 = \$47\,410.67$$

b. Use the TVM Solver to determine this new monthly payment.

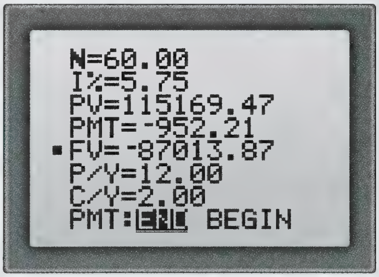


```
 N=180.00
 I%=5.75
 PV=115169.47
▪PMT=-952.21
 FV=0.00
 P/Y=12.00
 C/Y=2.00
 PMT:END BEGIN
```

= 15 × 12

Drew and Janine's new monthly payment is $952.21.

Use the TVM Solver to determine the amount owing on the mortgage after 5 years.



```
N=60.00
I%=5.75
PV=115169.47
PMT=-952.21
▪FV=-87013.87
P/Y=12.00
C/Y=2.00
PMT:END BEGIN
```

$= 5 \times 12$

Remember to re-enter this amount.

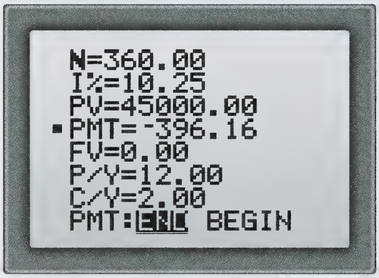
The amount owing after 5 years is $87 013.87.

Total paid = $952.21 × 60
= $57 132.60

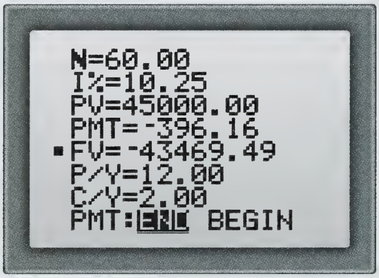
Difference in principal = $115 169.47 − $87 013.87
= $28 155.60

∴ Total interest paid = $57 132.60 − $28 155.60
= $28 977.00

7. Use the TVM Solver to find the monthly payments for a $45 000 mortgage, amortized over 30 years, at a rate of 10.25%; then determine the amount of principal left to pay after the first 5-year term. The following calculator displays show the needed calculations.



```
N=360.00
I%=10.25
PV=45000.00
▪PMT=-396.16
FV=0.00
P/Y=12.00
C/Y=2.00
PMT:END BEGIN
```

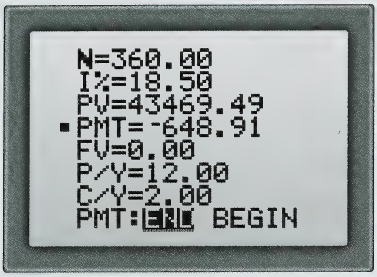


```
N=60.00
I%=10.25
PV=45000.00
PMT=-396.16
▪FV=-43469.49
P/Y=12.00
C/Y=2.00
PMT:END BEGIN
```

The principal remaining to be paid after the first 5 years is $43 469.49.

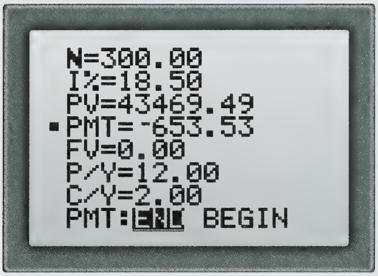
Now, determine the new monthly payments for various amortization periods with the higher 18.5% interest rate.

Over 30 years



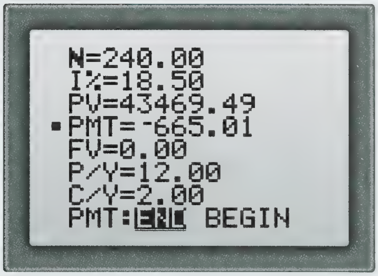
```
N=360.00
I%=18.50
PV=43469.49
▪PMT=-648.91
FV=0.00
P/Y=12.00
C/Y=2.00
PMT:END BEGIN
```

Over 25 years



```
N=300.00
I%=18.50
PV=43469.49
▪PMT=-653.53
FV=0.00
P/Y=12.00
C/Y=2.00
PMT:END BEGIN
```

Over 20 years



```
N=240.00
I%=18.50
PV=43469.49
▪PMT=-665.01
FV=0.00
P/Y=12.00
C/Y=2.00
PMT:END BEGIN
```

The couple would have to make considerably more to be able to qualify for a mortgage with $648.91 monthly payments, compared to the $396.16 payments they had been making. This is a 63.8% increase for an amortization period that did not change.

3. Textbook exercise "Communicating the Ideas," p. 190

Answers may vary. A sample answer is given.

Your gross income is $95 000 a year, or $7916.67 a month. You should not spend over 40% of your income on your house and other financial obligations. This works out to be $3166.67/month. If you subtract your credit-card payment of $200/month, your taxes of $265/month, and your heating costs of $62/month, you have $2639.67 you can put towards your mortgage. If you pay the maximum amount per month, your mortgage of $65 000 can be paid out in 27 months, saving you over $50 000 in interest you would have had to pay if you were to finance your mortgage over 20 years. The faster you pay your mortgage, the less interest you have to pay.

Activity 6: Renting Versus Buying

1. Textbook exercises 1.a. of "Practise Your Prior Skills," p. 191

1. a.

$$\begin{aligned}\text{Assessed value} &= 75\% \text{ of market value}\\ &= 0.75 \times \$130\ 000\\ &= \$97\ 500.00\end{aligned}$$

$$\begin{aligned}\therefore \text{ Annual property tax} &= \frac{\text{Assessed value} \times \text{Mill rate}}{1000}\\ &= \frac{\$97\ 500 \times 32.8}{1000}\\ &= \$3198.00\end{aligned}$$

$$\begin{aligned}\therefore \text{ Monthly property tax payment} &= \frac{\$3198.00}{12}\\ &= 266.50\end{aligned}$$

Activity 6 (continued)

2. Textbook exercises 1 to 4 of "Discussing the Ideas," p. 194

1. Answers will vary. A sample answer is given.

Some maintenance items that may occur around the home are as follows:

- painting ($1000–$3000)
- carpet cleaning ($100–$250)
- duct cleaning ($50–$150)
- landscaping
- watering, fertilizing, and mowing the lawn
- repair of fences
- staining trim and/or the deck
- minor plumbing repairs

2. Utility costs will vary depending on the cost of electricity and gas. A typical homeowner can expect to pay between $100 and $200 per month for natural gas and approximately $200 a month for electricity and water (including sanitation). Utility costs will vary from family to family depending on family size; conservation practices; insulation of the house; and efficiencies of the furnace, hot-water tank, and major appliances.

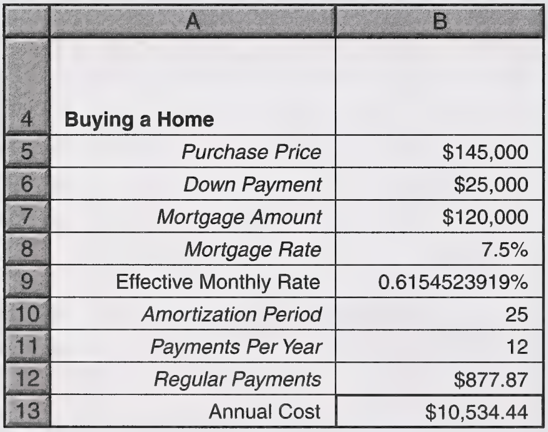
3. Some costs that are common to renters and homeowners may include telephone charges, cable costs, cost of electricity, cleaning expenses, and security charges.

4. Answers will vary. A sample answer is given.

Taxes are commonly charged on the fair market value of the property. In these jurisdictions the mill rate is typically between 5 mills and 10 mills. Education taxes are normally included on the bill and are approximately the same as the property taxes.

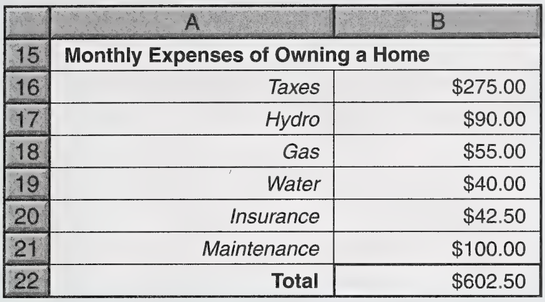
3. Textbook exercises 1 to 9 of "Investigation: Is It Better to Rent or to Buy?," pp. 192 and 193

1. and 2.



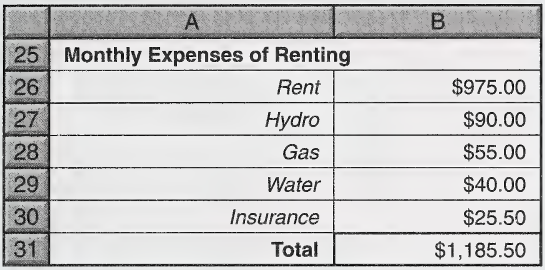
	A	B
4	**Buying a Home**	
5	*Purchase Price*	$145,000
6	*Down Payment*	$25,000
7	*Mortgage Amount*	$120,000
8	*Mortgage Rate*	7.5%
9	Effective Monthly Rate	0.6154523919%
10	*Amortization Period*	25
11	*Payments Per Year*	12
12	*Regular Payments*	$877.87
13	Annual Cost	$10,534.44

3.



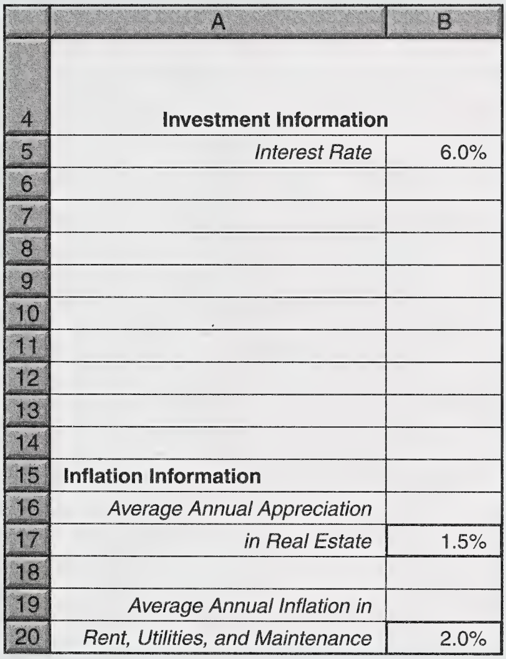
	A	B
15	**Monthly Expenses of Owning a Home**	
16	*Taxes*	$275.00
17	*Hydro*	$90.00
18	*Gas*	$55.00
19	*Water*	$40.00
20	*Insurance*	$42.50
21	*Maintenance*	$100.00
22	**Total**	$602.50

4.



	A	B
25	**Monthly Expenses of Renting**	
26	*Rent*	$975.00
27	*Hydro*	$90.00
28	*Gas*	$55.00
29	*Water*	$40.00
30	*Insurance*	$25.50
31	**Total**	$1,185.50

5.

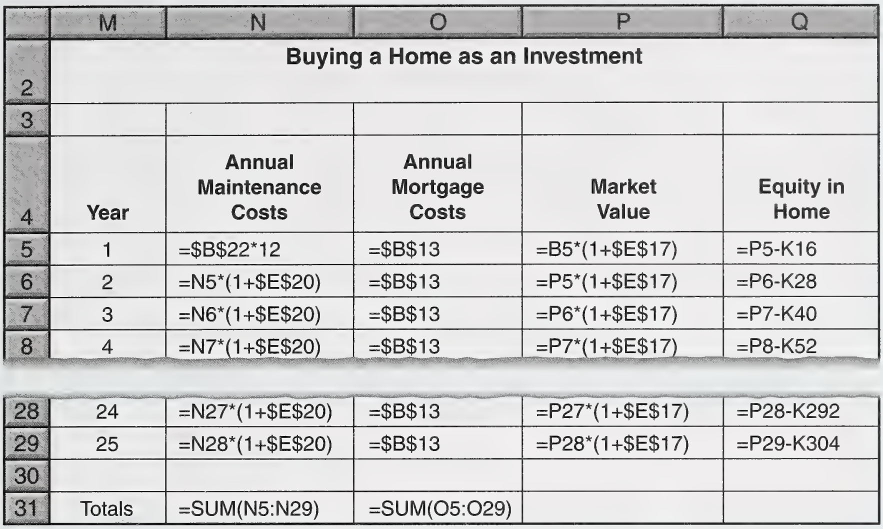


	A	B
4	**Investment Information**	
5	*Interest Rate*	6.0%
6		
7		
8		
9		
10		
11		
12		
13		
14		
15	**Inflation Information**	
16	*Average Annual Appreciation*	
17	*in Real Estate*	1.5%
18		
19	*Average Annual Inflation in*	
20	*Rent, Utilities, and Maintenance*	2.0%

6.

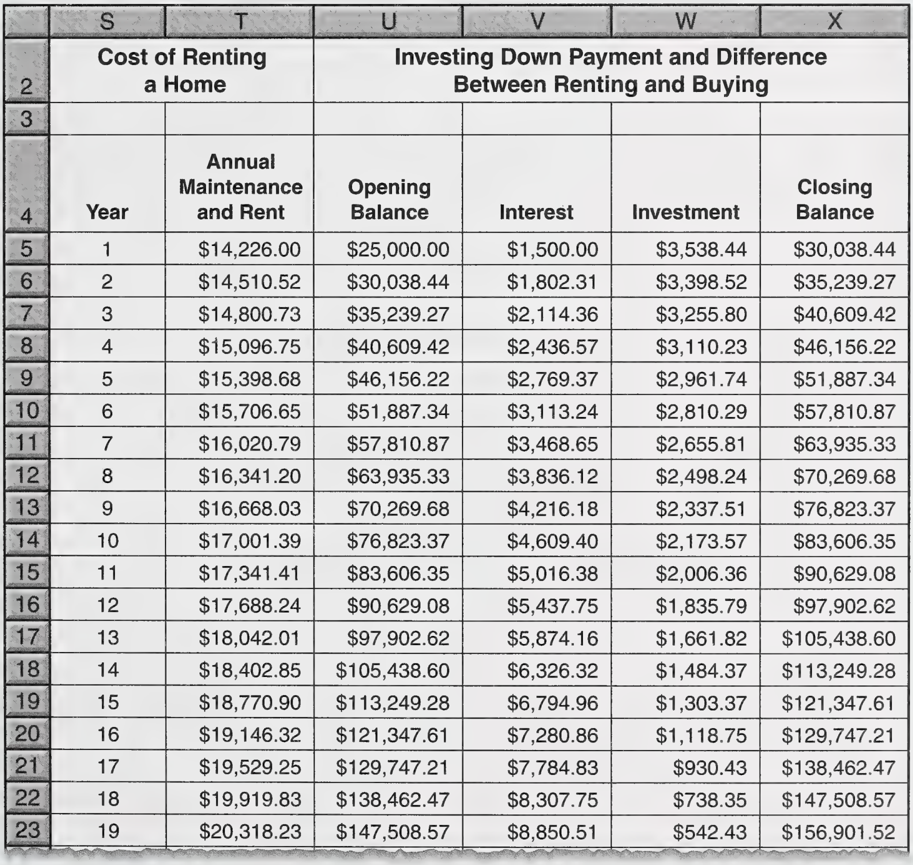
	M	N	O	P	Q
2	Buying a Home as an Investment				
3					
4	Year	Annual Maintenance Costs	Annual Mortgage Costs	Market Value	Equity in Home
5	1	$7,230.00	$10,534.44	$147,175.00	$28,904.70
6	2	$7,374.60	$10,534.44	$149,382.63	$32,974.18
7	3	$7,522.09	$10,534.44	$151,623.36	$37,219.03
8	4	$7,672.53	$10,534.44	$153,897.71	$41,650.63
9	5	$7,825.98	$10,534.44	$156,206.18	$46,281.16
10	6	$7,982.50	$10,534.44	$158,549.27	$51,123.74
11	7	$8,142.15	$10,534.44	$160,927.51	$56,192.44
12	8	$8,305.00	$10,534.44	$163,341.43	$61,502.39
13	9	$8,471.10	$10,534.44	$165,791.55	$67,069.82
14	10	$8,640.52	$10,534.44	$168,278.42	$72,912.18
15	11	$8,813.33	$10,534.44	$170,802.60	$79,048.23
16	12	$8,989.60	$10,534.44	$173,364.63	$85,498.11
17	13	$9,169.39	$10,534.44	$175,965.10	$92,283.48
18	14	$9,352.78	$10,534.44	$178,604.58	$99,427.60
19	15	$9,539.83	$10,534.44	$181,283.65	$106,955.50
20	16	$9,730.63	$10,534.44	$184,002.90	$114,894.07
21	17	$9,925.24	$10,534.44	$186,762.95	$123,272.21
22	18	$10,123.75	$10,534.44	$189,564.39	$132,121.01
23	19	$10,326.22	$10,534.44	$192,407.86	$141,473.89
24	20	$10,532.74	$10,534.44	$195,293.98	$151,366.79
25	21	$10,743.40	$10,534.44	$198,223.39	$161,838.33
26	22	$10,958.27	$10,534.44	$201,196.74	$172,930.08
27	23	$11,177.43	$10,534.44	$204,214.69	$184,686.73
28	24	$11,400.98	$10,534.44	$207,277.91	$197,156.34
29	25	$11,629.00	$10,534.44	$210,387.08	$210,390.61
30					
31	Totals	$231,579.07	$263,361.00		

The following is a partial spreadsheet showing the formulas used to calculate this area.



	M	N	O	P	Q
2	Buying a Home as an Investment				
3					
4	Year	Annual Maintenance Costs	Annual Mortgage Costs	Market Value	Equity in Home
5	1	=$B$22*12	=$B$13	=B5*(1+$E$17)	=P5-K16
6	2	=N5*(1+$E$20)	=$B$13	=P5*(1+$E$17)	=P6-K28
7	3	=N6*(1+$E$20)	=$B$13	=P6*(1+$E$17)	=P7-K40
8	4	=N7*(1+$E$20)	=$B$13	=P7*(1+$E$17)	=P8-K52
28	24	=N27*(1+$E$20)	=$B$13	=P27*(1+$E$17)	=P28-K292
29	25	=N28*(1+$E$20)	=$B$13	=P28*(1+$E$17)	=P29-K304
30					
31	Totals	=SUM(N5:N29)	=SUM(O5:O29)		

7.



	S	T	U	V	W	X
2	Cost of Renting a Home		Investing Down Payment and Difference Between Renting and Buying			
3						
4	Year	Annual Maintenance and Rent	Opening Balance	Interest	Investment	Closing Balance
5	1	$14,226.00	$25,000.00	$1,500.00	$3,538.44	$30,038.44
6	2	$14,510.52	$30,038.44	$1,802.31	$3,398.52	$35,239.27
7	3	$14,800.73	$35,239.27	$2,114.36	$3,255.80	$40,609.42
8	4	$15,096.75	$40,609.42	$2,436.57	$3,110.23	$46,156.22
9	5	$15,398.68	$46,156.22	$2,769.37	$2,961.74	$51,887.34
10	6	$15,706.65	$51,887.34	$3,113.24	$2,810.29	$57,810.87
11	7	$16,020.79	$57,810.87	$3,468.65	$2,655.81	$63,935.33
12	8	$16,341.20	$63,935.33	$3,836.12	$2,498.24	$70,269.68
13	9	$16,668.03	$70,269.68	$4,216.18	$2,337.51	$76,823.37
14	10	$17,001.39	$76,823.37	$4,609.40	$2,173.57	$83,606.35
15	11	$17,341.41	$83,606.35	$5,016.38	$2,006.36	$90,629.08
16	12	$17,688.24	$90,629.08	$5,437.75	$1,835.79	$97,902.62
17	13	$18,042.01	$97,902.62	$5,874.16	$1,661.82	$105,438.60
18	14	$18,402.85	$105,438.60	$6,326.32	$1,484.37	$113,249.28
19	15	$18,770.90	$113,249.28	$6,794.96	$1,303.37	$121,347.61
20	16	$19,146.32	$121,347.61	$7,280.86	$1,118.75	$129,747.21
21	17	$19,529.25	$129,747.21	$7,784.83	$930.43	$138,462.47
22	18	$19,919.83	$138,462.47	$8,307.75	$738.35	$147,508.57
23	19	$20,318.23	$147,508.57	$8,850.51	$542.43	$156,901.52

24	20	$20,724.60	$156,901.52	$9,414.09	$342.59	$166,658.20
25	21	$21,139.09	$166,658.20	$9,999.49	$138.75	$176,796.44
26	22	$21,561.87	$176,796.44	$10,607.79	-$69.16	$187,335.06
27	23	$21,993.11	$187,335.06	$11,240.10	-$281.23	$198,293.93
28	24	$22,432.97	$198.293.93	$11,897.64	-$497.55	$209,694.02
29	25	$22,881.63	$209,694.02	$12,581.64	-$718.19	$221,557.48
30						
31		$455,663.04				

The following is a partial spreadsheet showing the formulas used to calculate this area.

	S	T	U	V	W	X
2	Cost of Renting a Home		Investing Down Payment and Difference Between Renting and Buying			
3						
4	Year	Annual Maintenance and Rent	Opening Balance	Interest	Investment	Closing Balance
5	1	=B31*12	=B6	=U5*$E$5	=N5+O5-T5	=U5+V5+W5
6	2	=T5*(1+$E$20)	=X5	=U6*$E$5	=N6+O6-T6	=U6+V6+W6
7	3	=T6*(1+$E$20)	=X6	=U7$E$5	=N7+O7-T7	=U7+V7+W7
8	4	=T7*(1+$E$20)	=X7	=U8*$E$5	=N8+O8-T8	=U8+V8+W8
28	24	=T27*(1+$E$20)	=X27	=U28*$E$5	=N28+O28-T28	=U28+V28+W28
29	25	=T28*(1+$E$20)	=X28	=U29*$E$5	=N29+O29-T29	=U29+V29+W29
30						
31		=SUM(T5:T29)				

8. **a.** Answers will vary. Some considerations are as follows:

- Are real estate values likely to increase, remain steady, or decline?
- How permanent is your job?
- Are you likely to move to another city?
- Will your job move to another part of the city?
- Is the size of your family going to increase or decrease?
- Are you willing to put in the effort to maintain your home and yard?
- Do you plan to travel and be away for extended periods?

b. Since interest on investments is taxable, income taxes would decrease the accumulated value of investing the differences between owning and renting. (Some or all of the interest and investment money could be put into an RRSP to avoid paying income tax on it.)

9. Answers will vary. A sample answer is given.

From a financial point of view, buying a home is better. In 25 years they will be far better off owning their home. They will have spent $494 939 for a place to live and will have ownership of a $210 387.08 asset. The net cost for their living place will have been $284 549.46. If they had rented, they would have spent $455 663.04. Since a large income would be needed to acquire a $120 000 mortgage, their marginal tax rate would most likely be 44%. This would decrease their investment income to $124 072.19; so, the net cost of renting would be $331 590.85. This is over $40 000 more than buying, and the cost of living in their own home will be lower than renting once the mortgage is paid.

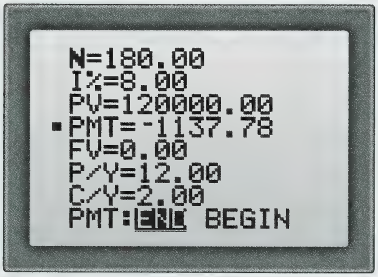
4. Textbook exercises 1 and 2 of "Exercises: Checking Your Skills," pp. 194 and 195

1. Determine Justin and Penny's property tax.

$$\begin{aligned}\text{Property tax} &= \frac{\text{Assessed Value} \times \text{Mill rate}}{1000}\\ &= \frac{(\$165\,000 \times 0.80) \times 36.4}{1000}\\ &= \frac{\$132\,000 \times 36.4}{1000}\\ &= \$4804.80\end{aligned}$$

$$\therefore \text{ Monthly tax payable} = \frac{\$4804.80}{12} = \$400.40$$

Use the TVM Solver to determine their monthly mortgage payment.

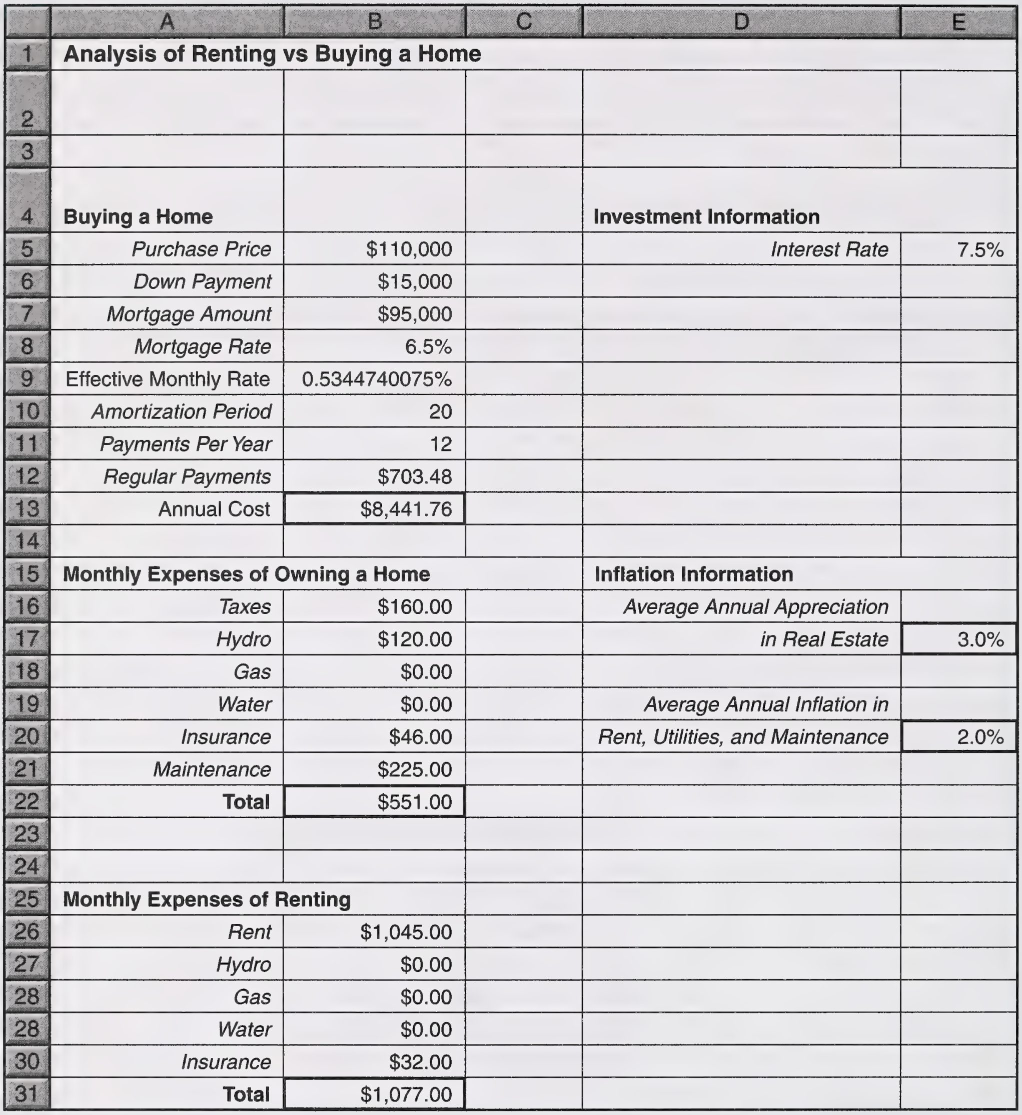


```
N=180.00
I%=8.00
PV=120000.00
▪PMT=-1137.78
FV=0.00
P/Y=12.00
C/Y=2.00
PMT:END BEGIN
```

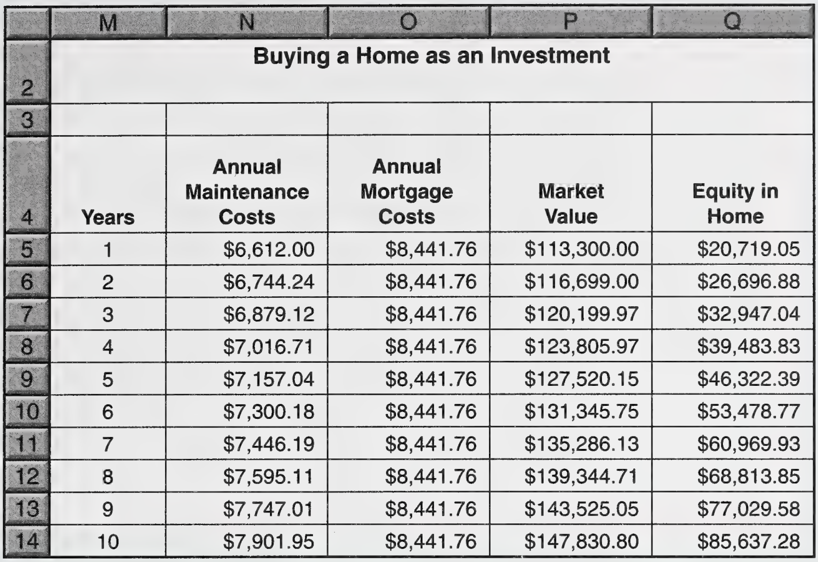
Justin and Penny's monthly mortgage payment is $1137.78.

$$\begin{aligned}\therefore \text{ Average monthly cost} &= \text{Mortgage} + \text{Property tax} + \text{Hydro} + \text{Gas} + \text{Water} + \text{Insurance} + \text{Maintenance}\\ &= \$1137.78 + \$400.40 + \$90.00 + \$62.00 + \frac{\$110.00}{3} + \frac{\$550}{12} + \frac{\$1150}{12}\\ &= \$1868.51\end{aligned}$$

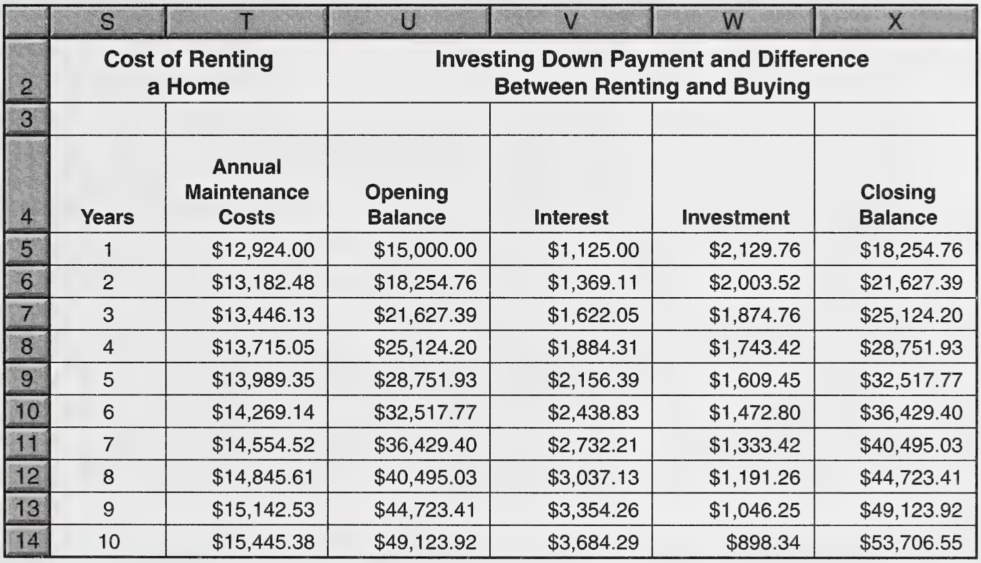
2. Use the spreadsheet *RENTBUY* to compare buying and renting. Enter the values shown.



	A	B	C	D	E
1	**Analysis of Renting vs Buying a Home**				
2					
3					
4	**Buying a Home**			**Investment Information**	
5	*Purchase Price*	$110,000		*Interest Rate*	7.5%
6	*Down Payment*	$15,000			
7	*Mortgage Amount*	$95,000			
8	*Mortgage Rate*	6.5%			
9	Effective Monthly Rate	0.5344740075%			
10	*Amortization Period*	20			
11	*Payments Per Year*	12			
12	*Regular Payments*	$703.48			
13	Annual Cost	$8,441.76			
14					
15	**Monthly Expenses of Owning a Home**			**Inflation Information**	
16	*Taxes*	$160.00		*Average Annual Appreciation*	
17	*Hydro*	$120.00		*in Real Estate*	3.0%
18	*Gas*	$0.00			
19	*Water*	$0.00		*Average Annual Inflation in*	
20	*Insurance*	$46.00		*Rent, Utilities, and Maintenance*	2.0%
21	*Maintenance*	$225.00			
22	**Total**	$551.00			
23					
24					
25	**Monthly Expenses of Renting**				
26	*Rent*	$1,045.00			
27	*Hydro*	$0.00			
28	*Gas*	$0.00			
28	*Water*	$0.00			
30	*Insurance*	$32.00			
31	**Total**	$1,077.00			



	M	N	O	P	Q
2	Buying a Home as an Investment				
3					
4	Years	Annual Maintenance Costs	Annual Mortgage Costs	Market Value	Equity in Home
5	1	$6,612.00	$8,441.76	$113,300.00	$20,719.05
6	2	$6,744.24	$8,441.76	$116,699.00	$26,696.88
7	3	$6,879.12	$8,441.76	$120,199.97	$32,947.04
8	4	$7,016.71	$8,441.76	$123,805.97	$39,483.83
9	5	$7,157.04	$8,441.76	$127,520.15	$46,322.39
10	6	$7,300.18	$8,441.76	$131,345.75	$53,478.77
11	7	$7,446.19	$8,441.76	$135,286.13	$60,969.93
12	8	$7,595.11	$8,441.76	$139,344.71	$68,813.85
13	9	$7,747.01	$8,441.76	$143,525.05	$77,029.58
14	10	$7,901.95	$8,441.76	$147,830.80	$85,637.28



	S	T	U	V	W	X
2	Cost of Renting a Home		Investing Down Payment and Difference Between Renting and Buying			
3						
4	Years	Annual Maintenance Costs	Opening Balance	Interest	Investment	Closing Balance
5	1	$12,924.00	$15,000.00	$1,125.00	$2,129.76	$18,254.76
6	2	$13,182.48	$18,254.76	$1,369.11	$2,003.52	$21,627.39
7	3	$13,446.13	$21,627.39	$1,622.05	$1,874.76	$25,124.20
8	4	$13,715.05	$25,124.20	$1,884.31	$1,743.42	$28,751.93
9	5	$13,989.35	$28,751.93	$2,156.39	$1,609.45	$32,517.77
10	6	$14,269.14	$32,517.77	$2,438.83	$1,472.80	$36,429.40
11	7	$14,554.52	$36,429.40	$2,732.21	$1,333.42	$40,495.03
12	8	$14,845.61	$40,495.03	$3,037.13	$1,191.26	$44,723.41
13	9	$15,142.53	$44,723.41	$3,354.26	$1,046.25	$49,123.92
14	10	$15,445.38	$49,123.92	$3,684.29	$898.34	$53,706.55

Based on financial considerations alone, it is better to buy the condominium than rent an apartment. After 10 years, their equity in the condo would be $85 637.28, but their investments would only be $53 706.55 if they rented.

5. Textbook exercise "Communicating the Ideas," p. 197

Answers will vary. A sample answer is given.

Some other factors that enter into the decision of whether home ownership is right for you are lifestyle considerations, employment prospects, maintenance concerns, resale value, personal security, and family concerns.

Module Review

1. Textbook exercise 1 of Part A of "What Should I Be Able to Do?," p. 201

1. a.

Age at Initial Investment	Number of Years to Invest	Investment Per Year	Future Value, if Growth Rate Is 6%	Future Value, if Growth Rate Is 10%	Future Value, if Growth Rate Is 14%
25	40	$1000.00	$154 761.97	$442 592.56	$1 342 025.10
30	35	$1142.86	$127 354.35	$309 742.91	$792 656.50
35	30	$1333.33	$105 410.65	$219 324.82	$475 714.61
40	25	$1600.00	$87 783.22	$157 355.30	$290 993.32
45	20	$2000.00	$73 571.18	$114 550.00	$182 049.86
50	15	$2666.67	$62 069.33	$84 726.72	$116 913.25
55	10	$4000.00	$52 723.18	$63 749.70	$77 349.18

b. Investing at age 25 instead of 30 will give you $27 407.62 more at a growth rate of 6%.
Investing at age 25 instead of 30 will give you $132 849.65 more at a growth rate of 10%.
Investing at age 25 instead of 30 will give you $549 368.60 more at a growth rate of 14%.

c. Investing at age 30 instead of 40 will give you $39 571.13 more at a growth rate of 6%.
Investing at age 30 instead of 40 will give you $152 387.61 more at a growth rate of 10%.
Investing at age 30 instead of 40 will give you $501 663.18 more at a growth rate of 14%.

d. The higher the growth rate, the greater the differences will be in the final amount of the investment.

2. Textbook exercises 2, 5, 6, 7, and 9 of Part B of "What Should I Be Able to Do?," pp. 201 to 204

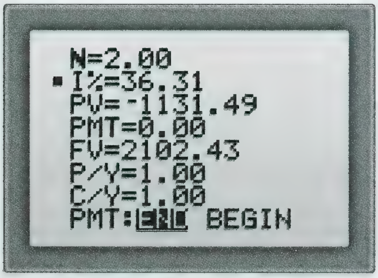
2. a.

Year	Opening Balance	Rate of Return	Closing Balance
1	$1000.00	12.1%	$1121.00
2	$1121.00	8.3%	$1214.04
3	$1214.04	(6.8%)	$1131.49
4	$1131.49	19.8%	$1355.53
5	$1355.53	55.1%	$2102.43

b. **For Years 4 and 5**

The opening balance for year 4 was $1131.49. The closing balance for year 5 was $2102.43.

Use the TVM Solver to determine the average compound rate of return. Enter the following values, and solve for I%.



```
  N=2.00
■ I%=36.31
  PV=-1131.49
  PMT=0.00
  FV=2102.43
  P/Y=1.00
  C/Y=1.00
  PMT:END BEGIN
```

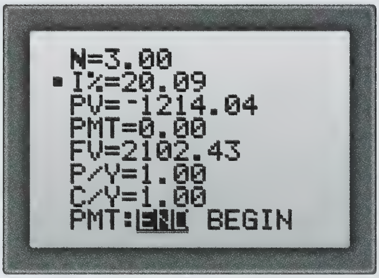
The average compound rate of return for the last 2 years was 36.31%.

Module Review (continued)

For Years 3, 4, and 5

The opening balance for year 3 was \$1214.04. The closing balance for year 5 was \$2102.43.

Use the TVM Solver to determine the average compound rate of return. Enter the following values, and solve for I%.



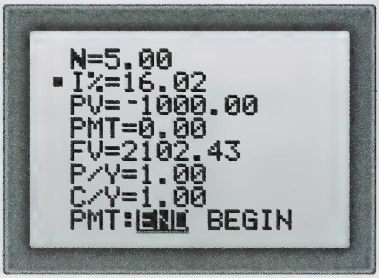
```
 N=3.00
▪I%=20.09
 PV=-1214.04
 PMT=0.00
 FV=2102.43
 P/Y=1.00
 C/Y=1.00
 PMT:END BEGIN
```

The average compound rate of return for the last 3 years was 20.09%.

For All 5 Years

The opening balance for year 1 was \$1000.00. The closing balance for year 5 was \$2102.43.

Use the TVM Solver to determine the average compound rate of return. Enter the following values, and solve for I%.



```
 N=5.00
▪I%=16.02
 PV=-1000.00
 PMT=0.00
 FV=2102.43
 P/Y=1.00
 C/Y=1.00
 PMT:END BEGIN
```

The compound rate of return for all 5 years was 16.02%.

5. **a.** For a person with a marginal tax rate of 25%, 75% of the interest that person earned is retained as net income.

$$\begin{aligned} \therefore \text{ Net interest rate} &= 0.75 \times \text{Interest rate} \\ &= 0.75r \end{aligned}$$

b. To determine the net amount of an unregistered investment, *P*, apply the following formula:

$$\begin{aligned}\text{Net amount} &= P + 0.75rP \\ &= P(1+0.75r)\end{aligned}$$

c.

Investment Amount	Interest Rate	Marginal Tax Rate	Net Interest Rate	Number of Years of Investment	Total Value of Investment
$1000	12%	25%	9%	10	$2367.36
$2500	8.5%	39%	5.185%	15	$5336.39
$4000	14.5%	44%	8.12%	25	$28 165.08

6.

Investment	Amount	Average Return	Final Amount
Bank account	$2 400	3%	$2 472.00
Term deposit	$4 800	6.25%	$5 100.00
Guaranteed investment certificate	$12 500	8.5%	$13 562.50
Guaranteed investment certificate	$8 000	9.25%	$8 740.00
Mutual fund	$42 000	28.75%	$54 075.00
TOTALS	$69 700		$83 949.50

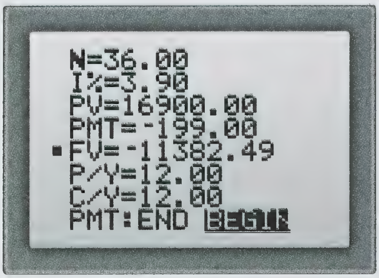
The increase over the amount invested is $\$83\,949.50 - \$69\,700.00 = \$14\,249.50$.

$$\begin{aligned}\therefore \text{ Average rate of return} &= \frac{14\,249.50}{69\,700.00} \times 100\% \\ &\doteq 20.4\%\end{aligned}$$

7. **a.** The lease prices might be different because of different interest rates or different residual values.

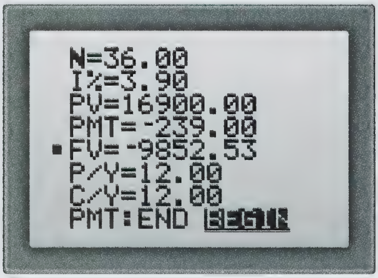
b. Use your TVM Solver to determine the residual value (or future value) of each car. Enter the following values and solve for FV.

Car A



```
N=36.00
I%=3.90
PV=16900.00
PMT=-199.00
■FV=-11382.49
P/Y=12.00
C/Y=12.00
PMT:END BEGIN
```

Car B



```
N=36.00
I%=3.90
PV=16900.00
PMT=-239.00
■FV=-9852.53
P/Y=12.00
C/Y=12.00
PMT:END BEGIN
```

Remember: For leases, change the payments to the beginning of the month.

The residual value of Car A is $11 382.49, and the residual value of Car B is $9852.53.

9. **a.** Heather can spend 40% of her gross income on debts servicing and housing.

$$\begin{aligned}\text{Monthly income} &= \frac{\$38\,000}{12}\\ &= \$3166.67\end{aligned}$$

$$\begin{aligned}\therefore\ 40\%\ \text{of monthly income} &= 0.40 \times \$3166.67\\ &= \$1266.67\end{aligned}$$

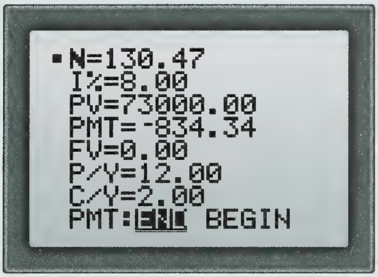
From this, subtract her car lease payment, monthly property taxes, and heating bills.

$$\begin{aligned}\text{Maximum allowable mortgage payment} &= \$1266.67\left(\$189.00 + \frac{\$2200}{12} + \$60.00\right)\\ &= \$1266.67 - \$432.33\\ &= \$834.34\end{aligned}$$

b. Use the TVM Solver to determine the shortest amortization period for her.

$$\begin{aligned}\text{Mortgage} &= \$88\,000 - \$15\,000\\ &= \$73\,000\end{aligned}$$

Enter the following values, and solve for N.



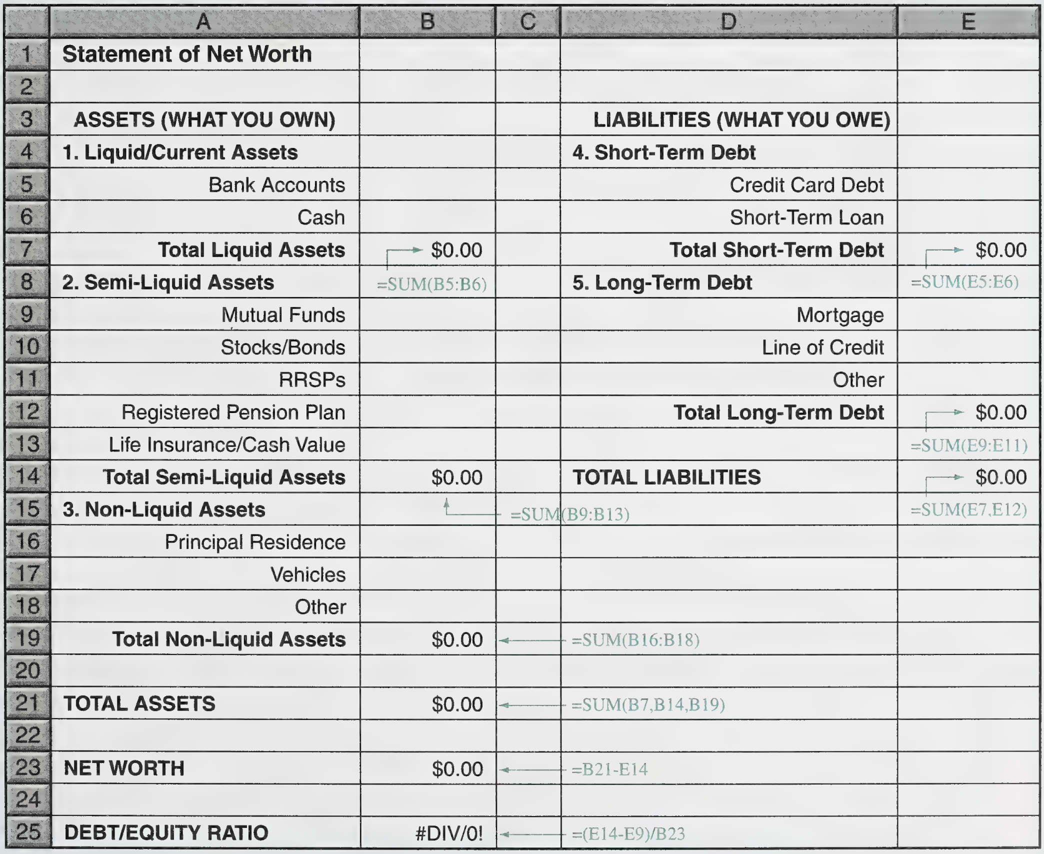
```
▪N=130.47
 I%=8.00
 PV=73000.00
 PMT=-834.34
 FV=0.00
 P/Y=12.00
 C/Y=2.00
 PMT:END BEGIN
```

The shortest amortization period is 131 months or one month less than 11 years.

Enrichment

1. Textbook exercises 3 and 4 of Part B of "What Should I Be Able to Do?" pp. 202 and 203

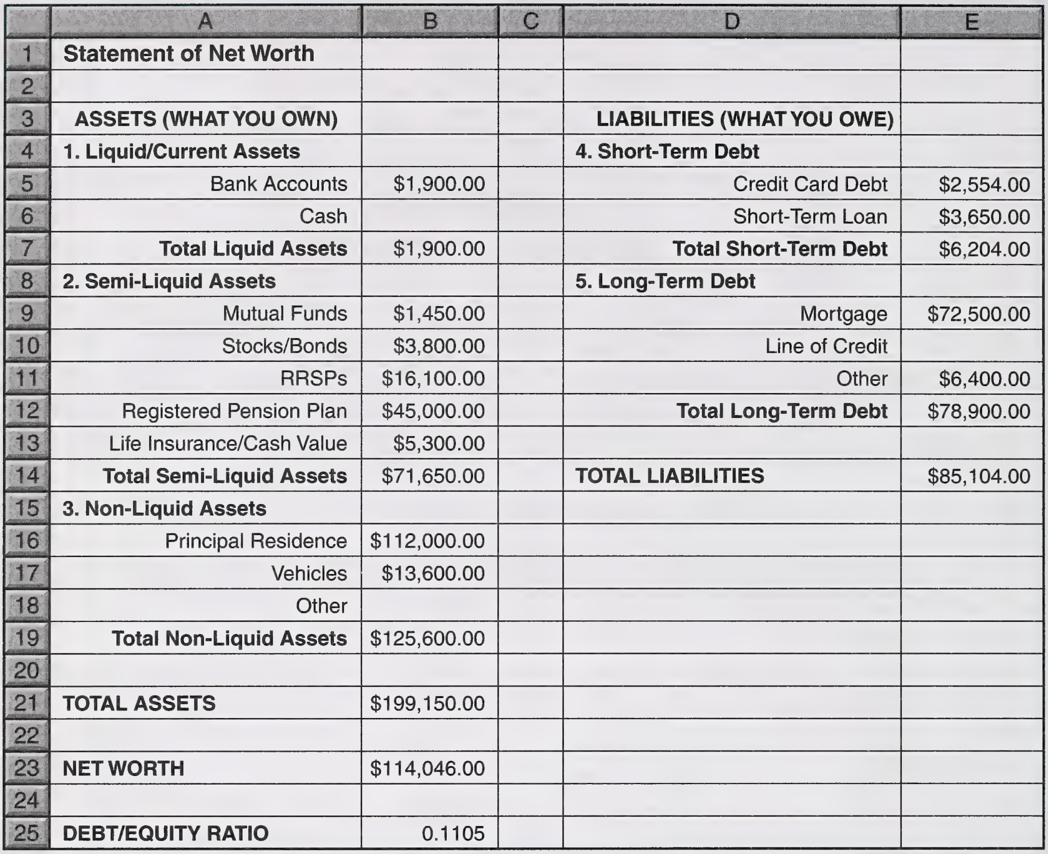
3. Your spreadsheet may look somewhat like the following.



	A	B	C	D	E
1	**Statement of Net Worth**				
2					
3	**ASSETS (WHAT YOU OWN)**			**LIABILITIES (WHAT YOU OWE)**	
4	**1. Liquid/Current Assets**			**4. Short-Term Debt**	
5	Bank Accounts			Credit Card Debt	
6	Cash			Short-Term Loan	
7	**Total Liquid Assets**	$0.00		**Total Short-Term Debt**	$0.00
8	**2. Semi-Liquid Assets**	=SUM(B5:B6)		**5. Long-Term Debt**	=SUM(E5:E6)
9	Mutual Funds			Mortgage	
10	Stocks/Bonds			Line of Credit	
11	RRSPs			Other	
12	Registered Pension Plan			**Total Long-Term Debt**	$0.00
13	Life Insurance/Cash Value				=SUM(E9:E11)
14	**Total Semi-Liquid Assets**	$0.00		**TOTAL LIABILITIES**	$0.00
15	**3. Non-Liquid Assets**		=SUM(B9:B13)		=SUM(E7,E12)
16	Principal Residence				
17	Vehicles				
18	Other				
19	**Total Non-Liquid Assets**	$0.00		=SUM(B16:B18)	
20					
21	**TOTAL ASSETS**	$0.00		=SUM(B7,B14,B19)	
22					
23	**NET WORTH**	$0.00		=B21-E14	
24					
25	**DEBT/EQUITY RATIO**	#DIV/0!		=(E14-E9)/B23	

Module Review (continued)

4.



	A	B	C	D	E
1	**Statement of Net Worth**				
2					
3	**ASSETS (WHAT YOU OWN)**			**LIABILITIES (WHAT YOU OWE)**	
4	**1. Liquid/Current Assets**			**4. Short-Term Debt**	
5	Bank Accounts	$1,900.00		Credit Card Debt	$2,554.00
6	Cash			Short-Term Loan	$3,650.00
7	**Total Liquid Assets**	$1,900.00		**Total Short-Term Debt**	$6,204.00
8	**2. Semi-Liquid Assets**			**5. Long-Term Debt**	
9	Mutual Funds	$1,450.00		Mortgage	$72,500.00
10	Stocks/Bonds	$3,800.00		Line of Credit	
11	RRSPs	$16,100.00		Other	$6,400.00
12	Registered Pension Plan	$45,000.00		**Total Long-Term Debt**	$78,900.00
13	Life Insurance/Cash Value	$5,300.00			
14	**Total Semi-Liquid Assets**	$71,650.00		**TOTAL LIABILITIES**	$85,104.00
15	**3. Non-Liquid Assets**				
16	Principal Residence	$112,000.00			
17	Vehicles	$13,600.00			
18	Other				
19	**Total Non-Liquid Assets**	$125,600.00			
20					
21	**TOTAL ASSETS**	$199,150.00			
22					
23	**NET WORTH**	$114,046.00			
24					
25	**DEBT/EQUITY RATIO**	0.1105			

2. Because a mortgage is a large proportion of most people's debt, a better indication of their borrowing habits will focus on debt other than their outstanding mortgage.

Module Project: Lease or Buy a Car?

Completing the Project

The "Financing Your Car" part of the student answers is well done. It is neat and clearly laid out with all calculations shown. There is one minor error, however, in that the payment is listed as $254; it should be listed as $252. The context for each part might be made more clear so there would be no need to go back to the questions to follow the work.

Image Credits

Cover and Title Page

collage: PhotoDisc Collection/Getty Images

Welcome Page

PhotoDisc Collection/Getty Images

Page	
2	**collage:** Image Club StudioGear/EyeWire Collection/Getty Images (top and right), PhotoDisc Collection/Getty Images (bottom left)
4	PhotoDisc Collection/Getty Images
7	EyeWire Collection/Getty Images
8	**collage:** PhotoDisc Collection/Getty Images, Image Club StudioGear/EyeWire Collection/Getty Images
9	**collage:** ©2001–2002 www.arttoday.com (car lot)
10	©2001–2002 www.arttoday.com
11	**top:** ©2001–2002 www.arttoday.com **bottom:** PhotoDisc Collection/Getty Images
12	PhotoDisc Collection/Getty Images
13	**bottom:** Image Club StudioGear/EyeWire Collection/Getty Images
15	Image Club StudioGear/EyeWire Collection/Getty Images
16	PhotoDisc Collection/Getty Images
19	PhotoDisc Collection/Getty Images
21	Corel Corporation
22	Image Club StudioGear/EyeWire Collection/Getty Images
23	Image Club StudioGear/EyeWire Collection/Getty Images
30	**top:** Image Club StudioGear/EyeWire Collection/Getty Images **bottom:** PhotoDisc Collection/Getty Images
31	Corel Corporation
32	©2001–2002 www.arttoday.com
33	PhotoDisc Collection/Getty Images
34	©2001–2002 www.arttoday.com
35	Image Club StudioGear/EyeWire Collection/Getty Images
38	Image Club StudioGear/EyeWire Collection/Getty Images
40	**collage:** ©2001–2002 www.arttoday.com **bottom:** ©2001–2002 www.arttoday.com
41	©2001–2002 www.arttoday.com
43	EyeWire Collection/Getty Images
44	Image Club StudioGear/EyeWire Collection/Getty Images
45	PhotoDisc Collection/Getty Images
49	PhotoDisc Collection/Getty Images
51	Image Club StudioGear/EyeWire Collection/Getty Images
52	**top:** Image Club StudioGear/EyeWire Collection/Getty Images **bottom:** PhotoDisc Collection/Getty Images
53	PhotoDisc Collection/Getty Images
55	Image Club StudioGear/EyeWire Collection/Getty Images
56	©2001–2002 www.arttoday.com
57	©2001–2002 www.arttoday.com
58	**top:** EyeWire Collection/Getty Images **bottom:** PhotoDisc Collection/Getty Images
59	**collage:** PhotoDisc Collection/Getty Images

ILL

© 2002